



Comptroller of the Currency
Administrator of National Banks

Construction and Validation of Econometric Models for Ultimate LGD on Bonds and Loans

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Outline

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Motivation

- ❑ Loss Given Default (LGD) – ultimate economic loss per dollar of outstanding balance at default (or one minus the Recovery Rate)
- ❑ LGD is a critical parameter in various facets of credit risk modeling – expected loss /allowance, pricing, capital
- ❑ Basel II Internal Ratings Based (IRB) advanced approach to regulatory credit capital requires banks to estimate LGD
- ❑ May be measured either on a nominal (undiscounted) or economic (discounted) basis – we care about the latter
- ❑ Here we measure the market values of instruments received in settlement of financial distress, a proxy for true economic LGD

Background and Measurement Issues

- ❑ Choice of discount rate – risk free vs. risk adjusted?
- ❑ The definition of default - bankruptcy vs. broader concept?
- ❑ Unit of observations - obligor (firm or estate) vs. instrument (facility)
- ❑ “Actuarial” approach (discount cash flows) vs. market for distressed debt (trading or settlement prices)
- ❑ Consistency with other credit risk parameters - Exposure at Default (EAD) and Probability of Default (PD)

Background and Measurement Issues

(continued)

- ❑ Many extant credit risk models assume LGD to be fixed despite evidence it is stochastic and predictable with respect to other variables

- ❑ The boundedness of LGD gives rise to unique statistical issues – boundary bias

- ❑ Determinants of ultimate LGD considered here and elsewhere:
 - ❑ Contractual features – collateral, seniority, debt cushion, facility type
 - ❑ Capital structure: % secured / bank debt, number creditor classes
 - ❑ Borrower – profitability, industry, liquidity, size, leverage
 - ❑ Systematic factors – macroeconomic state, debt market
 - ❑ LGD at default
 - ❑ Cumulative abnormal equity returns

Synopsis and Major Conclusions

- ❑ Empirical study of LGD with sample of agency rated firms (1985-2004) – most major U.S. defaults
- ❑ Alternative econometric models: Beta Link Generalized Linear (BLGLM), Kullback-Leibler Relative Entropy (KLRE), Generalized Beta Kernel Density (BDKE)
 - ❑ In single equation models, BLGLM outperforms linear regression on a transformed LGD variable
- ❑ Model LGD at the firm and instrument level, separately and jointly
- ❑ Consideration both explanatory variables - a macroeconomic factor, equity returns and the price of traded debt at default
- ❑ Validation through resampling out-of-time and out-of-sample

Synopsis and Major Conclusions (continued)

- ❑ Feedback effect between ultimate obligor and instrument LGD

- ❑ Demonstrate the significance of debt and equity market determinants
 - ❑ Price of instrument debt at default
 - ❑ Cumulative abnormal returns on equity prior to default

- ❑ Importance of firm specific financial ratios
 - ❑ Greater leverage, tangibility, market valuation, cash flow or liquidity associated with lower ultimate LGD

- ❑ Evidence that firms investment grade at origination have significantly lower ultimate LGDs

- ❑ Macroeconomic variables on firm LGD: aggregate default rates, equity market returns, dummy variables for 2000-2002 downturn

Synopsis and Major Conclusions (continued)

- ❑ Evidence of an industry distress effect (profitability) at the obligor level

- ❑ Contractual features : better ranking of collateral, less (more) debt above (below) & higher seniority of creditor class implies lower LGD

- ❑ Capital structure variables matter at the obligor level: number of creditor classes, proportions of secured and bank debt are all significantly and inversely related to the ultimate LGD

- ❑ Parametric model (BLGLM) performs worse (better) in- (out-of) sample for single equation (obligor or facility)

Literature Review: Theoretical Models

- ❑ Structural models: Merton(1974), Black and Cox (1976), Geske(1977), Vasicek(1984), Kim et al (1993), Hull & White (1995), Longstaff & Schwartz (1995)
 - ❑ Endogenous PD but LGD not independently modeled

- ❑ Reduced form models: Litterman & Iben (1991), Madan & Unal (1995), Jarrow & Turnbull (1995), Jarrow et al (1997), Lando (1998), Duffie & Singleton (1999), Duffie (1998)
 - ❑ LGD is exogenous but may be correlated with PD process

- ❑ Credit VaR models: Creditmetrics™ (Gupton et al, 1997), KMV™
 - ❑ Typically models LGD as exogenous but stochastic

- ❑ Hybrid approaches: Frye (2000), Jarrow (2001), Jokivuolle et al (2003), Carey & Gordy (2003), Bakshi et al (2001)
 - ❑ Realistic LGD assumptions: correlation with PD & systematic factors

Literature Review: Empirical Studies

- ❑ Bond market studies: Altman & Kishore (1996), Altman & Eberhart (1994) & Fridson et al (Merrill Lynch 2001)
- ❑ Bank studies: Altman, Haldeman and Narayanan (1977), Citigroup (Asarnow & Edwards, 1995), JP Morgan Chase (Araten et al, 2003)
- ❑ Rating agencies: Moody's (Hamilton et al, 2001), S&P (Keisman et al, 2002) , Moody's (Cantor et al, 2004)
- ❑ Bank consortiums: Loan Pricing Corporation (2001), Risk Management Association (2000)
- ❑ Vendor Models: S&P LossStats™ (Friedman & Sandow, 2003), Moody's LoosCalc2™ (Gupton, 2005)

Literature Review: Empirical Studies (continued)

Various recent academic studies have appeared on this topic:

- ❑ Hu and Perraudin (2002) – LGD/ PD correlation
- ❑ Renault and Scalliet (2003) – beta kernel density estimation
- ❑ Acharya et al (2004) - Industry distress (Shleifer & Vishny (2003) hypothesis – “fire-sale” effect)
- ❑ Altman (2005) – debt market supply/demand
- ❑ Carey & Gordy (2007) – estate level LGD and the role of bank debt
- ❑ Mason et al (2006) – option pricing & return on defaulted debt

Econometric Modeling of LGD I – Beta Link Generalized Linear Model (BLGLM)

- ❑ The GLM framework subsumes many of the models in the classical literature on limited / qualitative dependent variables
- ❑ GLMs long used in PD estimation (logistic regression) can be adapted for the LGD setting
- ❑ A specialization suitable for LGD estimation – modeling the link function with a mixture of beta distributions (Mallick and Gelfand, 1994)
- ❑ Allow the generalization of this in which the beta parameters are replaced by smooth, invertible functions of the linear predictors
- ❑ While in most cases no closed-form or analytic solution, but we may estimate the underlying parameters consistently and efficiently by maximizing (numerically) a log-likelihood function

Econometric Modeling of LGD II – Kullback-Liebler Relative Entropy (KLRE)

- ❑ The S&P LossCalc™ model is based upon this framework (Friedman and Sandow, 2003)
- ❑ Addresses boundary bias, multi-modality, non-normality, non-linearities, noisy data and over-fitting
- ❑ KLRE is a special case of information theoretic *maximum entropy inference* (MEI), a form of *minimum discrepancy estimator* (MDE)
- ❑ Criterion: model expected log differences between model & prior probability measures subject to degree of consistency to data
 - ❑ Single parameter family of distributions on frontier from data to prior
 - ❑ Globally convex, but infinite dimensional & non-standard constraint
- ❑ Under log utility, dual is finite dimensional concave maximization
 - ❑ Equivalent to MLE for exponential random variable

Econometric Modeling of LGD III – Beta Kernel Conditional Density Estimation (BKDE)

- ❑ Standard non-parametric estimators of unknown probability distribution functions typically utilize the Gaussian kernel
- ❑ Boundary bias problem: assigns non-zero density outside the support on dependent variable when smoothing near boundary.
- ❑ Chen (1999): beta kernel density estimator (BKDE) on $[0,1]$
- ❑ Properties: flexible functional form, bounded support, tractability, non-negativity & finite sample optimal convergence rate
 - ❑ Even if true density is unbounded at boundaries BKDE is consistent
- ❑ We extend the BKDE to a generalized BKDE (GBKDE): density a function of several variables affecting the smoothing
- ❑ Independent kernel & smoothing parameter in each dimension

Measures of Model Performance

- ❑ There exist two notions of accuracy in credit risk modeling:
 - ❑ *Discriminatory Accuracy (DA)* - rank order risk
 - ❑ *Predictive Accuracy (PA)* - forecast cardinal measures of risk

- ❑ We look at the KS statistic and the Area Under the Receiver Operating Curve (AUROC) to measure DA
 - ❑ Actually a quasi-AUROC derived from a Spearman Rank Order Correlation, since LGD is a continuous variable

- ❑ For PA look at Hoshmer-Lemeshow Chi-Squared (HLX2)
 - ❑ For GLMs in which the errors may not be normal, standard goodness-of-fit measure are difficult to interpret

- ❑ We perform an out-of-sample & out-of-time validation of PA / DA with a resampling (bootstrap) procedure, which yields distributions of the KS and AUROC

Database of Defaulted Loans and Bonds

- ❑ Merged Moody's Ultimate LGD Database™ Version 3.2 (June 2007 Release) & various public sources
- ❑ 871 (3902) defaulted firms (instruments) 1985-2006, U.S. large corporate, generally rated and public
- ❑ Instruments detailed by contractual features & capital structure:
 - ❑ Facility type & seniority
 - ❑ Collateral type and ranking (not values)
 - ❑ Position in the capital structure (debt above/below)
 - ❑ Original and defaulted amount
 - ❑ Out-of-court vs. bankruptcy, liquidations vs. reorganization
 - ❑ Instrument price or value of securities at resolution
- ❑ For some, price of traded debt, equity prices or financial statement data, at the time of default
- ❑ Discount using coupon rate at default (results not sensitive to alternatives)

Table 1 - Characteristics of LGD Observations by OCC QRF, 2007 Default Type and Availability of Financial Statement Data (S&P and Moody's Rated Defaults 1985-2006)

		Compustat					Non-Compustat					Total					
		LGD at Default ¹	Discounted LGD ²	Number of Creditor Classes ³	Principal at Default ⁴	Time-to-Resolution ⁵	LGD at Default	Discounted LGD	Number of Creditor Classes	Principal at Default	Time-to-Resolution	LGD at Default	Discounted LGD	Number of Creditor Classes	Principal at Default	Time-to-Resolution	
1.1 - Instrument Level Observations	Bankruptcy	Count	845	2736			151	537			996	3273					
		Average	63.17%	48.71%	2.4174	142,901	1.6644	61.31%	46.68%	2.4246	119,196	1.2928	62.89%	48.38%	2.4186	139,012	1.6035
		Stda.Dev.	28.66%	39.45%	0.8529	233,622	1.2676	27.07%	42.46%	1.0835	155,955	0.8114	28.42%	39.96%	0.8946	222,897	1.2124
		Minimum	-12.00%	-107.20%	1.0000	10	0.0028	2.52%	-124.19%	1.0000	10	0.0472	-12.00%	-124.19%	1.0000	10	0.0028
	Maximum	99.80%	100.00%	6.0000	4,600,000	9.1806	99.75%	100.00%	5.0000	1,225,000	4.9917	99.80%	100.00%	6.0000	4,600,000	9.1806	
	Out-of-Court	Count	106	519			16	110			122	629					
		Average	45.15%	17.49%	2.5010	172,251	0.2057	51.04%	10.76%	2.7909	103,577	0.2551	45.92%	16.31%	2.5517	160,241	0.2143
		Stda.Dev.	26.89%	30.84%	0.8763	307,473	0.5616	27.74%	29.13%	1.3484	166,368	0.5122	26.96%	30.63%	0.9804	288,905	0.5532
		Minimum	-1.00%	-70.89%	1.0000	10	0.0028	-7.87%	-99.35%	1.0000	435	0.0028	-7.87%	-99.35%	1.0000	10	0.0028
	Maximum	98.00%	100.00%	5.0000	3,000,000	5.6444	97.47%	99.00%	6.0000	1,350,000	1.9472	98.00%	100.00%	6.0000	3,000,000	5.6444	
	Total	Count	951	3255			167	647			1118	3902					
		Average	61.16%	43.73%	2.4307	147,581	1.4318	60.32%	40.57%	2.4869	116,540	1.1163	61.04%	43.21%	2.4400	142,434	1.3795
		Stda.Dev.	29.01%	39.88%	0.8570	247,061	1.2985	27.22%	42.68%	1.1401	157,750	0.8618	28.74%	40.37%	0.9101	234,883	1.2422
		Minimum	-12.00%	-107.20%	1.0000	10	0.0028	-7.87%	-124.19%	1.0000	10	0.0028	-12.00%	-124.19%	1.0000	10	0.0028
	Maximum	99.80%	100.00%	6.0000	4,600,000	9.1806	99.75%	100.00%	6.0000	1,350,000	4.9917	99.80%	100.00%	6.0000	4,600,000	9.1806	
	1.2 - Obligor Level Observations	Bankruptcy	Count	314	579			78	149			392	728				
Average			65.37%	50.95%	2.1969	675,099	1.5485	64.00%	53.24%	2.0940	429,584	1.3856	65.10%	51.41%	2.1758	624,849	1.5485
Stda.Dev.			23.85%	30.60%	0.8356	1,806,477	1.0810	21.80%	29.78%	1.0223	696,960	0.8820	23.44%	30.43%	0.8773	1,643,396	1.0810
Minimum			-5.52%	-89.65%	1.0000	1,979	0.0556	9.29%	-12.07%	1.0000	17,611	0.0889	-5.52%	-89.65%	1.0000	1,979	0.0556
Maximum		99.00%	100.00%	6.0000	32,279,012	6.8667	98.29%	99.88%	5.0000	6,415,738	4.9917	99.00%	100.00%	6.0000	32,279,012	6.8667	
Out-of-Court		Count	56	117			12	26			68	143					
		Average	53.82%	25.13%	2.2821	764,086	0.4293	55.15%	17.80%	2.4231	438,208	0.3822	54.06%	23.80%	2.3077	704,836	0.4208
		Stda.Dev.	25.38%	28.22%	0.8494	2,013,300	0.7877	21.50%	29.23%	1.1375	537,950	0.5230	24.60%	28.45%	0.9056	1,847,114	0.7452
		Minimum	5.29%	-64.89%	1.0000	14,495	0.0028	21.00%	-47.02%	1.0000	50,000	0.0028	5.29%	-64.89%	1.0000	14,495	0.0028
Maximum		92.77%	94.01%	5.0000	19,177,444	5.6444	97.47%	50.99%	4.0000	2,375,000	1.9472	97.47%	94.01%	5.0000	19,177,444	5.6444	
Total		Count	370	696			90	175			460	871					
		Average	63.62%	46.61%	2.2112	690,058	1.3952	62.82%	47.97%	2.1429	430,865	1.2365	63.46%	46.88%	2.1975	637,981	1.3633
		Stda.Dev.	24.41%	31.70%	0.8379	1,845,329	1.1584	21.85%	32.20%	1.0434	673,406	0.9105	23.91%	31.79%	0.8828	1,680,742	1.1144
		Minimum	-5.52%	-89.65%	1.0000	1,979	0.0028	-9.29%	47.02%	1.0000	17,611	0.0028	-5.52%	-89.65%	1.0000	1,979	0.0028
Maximum		99.00%	100.00%	6.0000	32,279,012	6.8667	98.29%	99.88%	5.0000	6,415,738	4.9917	99.00%	100.00%	6.0000	32,279,012	6.8667	

Table 1 - Characteristics of LGD Observations: Discussion

- ❑ About 80% firms on Compustat across default types and 83% bankruptcies across Compustat match or not
- ❑ Average estate (facility) principal at default \$638M (\$142M), slightly (much) more for bankruptcies (out-of-court)
- ❑ Mean LGD slightly higher at the obligor level (46.9% vs. 43.2%)
- ❑ LGD much higher for bankruptcies vs. out-of-court, 51.4% vs. 23.8% (48.4% vs. 16.3%) for obligors (instruments)
- ❑ LGDs at default are significantly higher than ultimate, 63.5% vs. 46.9% (62.9% vs. 48.4%) at the obligor (instrument) level
- ❑ Avg. time-to-resolution is 1.4 yrs, much less for out-of-court (0.4 yrs) & about 2 major creditor classes, slightly less for bankruptcies

Table 2 - LGD, Dollar Loss, Duration and Court Filing of Defaulted Instruments and Obligors by Cohort Year (S&P and Moody's Rated Defaults 1985-2006)

Year	Instruments					Obligors				
	Number of Defaults	Average Discounted LGD ¹	Total Defaulted Amount (\$MM) ²	Average Time-to-Resolution (Yrs.) ³	Proportion of Bankruptcy Filings	Number of Defaults	Average Discounted LGD ¹	Total Defaulted Amount ²	Average Time-to-Resolution ³	Proportion of Bankruptcy Filings
1985	2	82.70%	486	5.8653	100.00%	1	82.59%	486	5.9278	100.00%
1986	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
1987	43	51.86%	2,737	1.1458	39.53%	9	48.57%	2,907	1.6454	55.56%
1988	95	45.28%	5,008	2.3487	85.26%	21	56.22%	5,532	1.9657	76.19%
1989	99	59.88%	10,618	1.9773	74.75%	25	55.69%	11,247	1.3807	68.00%
1990	232	41.40%	26,860	1.6457	77.59%	63	41.48%	32,306	1.8244	77.78%
1991	305	34.75%	30,061	1.4700	70.16%	70	40.10%	28,661	1.5395	74.29%
1992	126	41.07%	10,783	1.5957	80.95%	27	45.67%	7,939	1.4727	85.19%
1993	121	34.33%	7,740	0.9837	82.64%	27	37.39%	5,516	1.2214	74.07%
1994	67	27.38%	4,736	0.9135	74.63%	26	42.26%	4,661	1.1594	80.77%
1995	116	32.51%	9,169	2.2478	96.55%	38	43.12%	9,235	1.6298	94.74%
1996	80	35.17%	5,684	1.1261	93.75%	24	42.37%	6,752	1.3885	91.67%
1997	67	39.62%	7,045	1.4865	95.52%	17	50.21%	6,138	1.3114	94.12%
1998	79	60.81%	8,258	1.3629	98.73%	31	57.28%	9,597	1.4180	96.77%
1999	208	43.85%	30,686	1.3268	87.98%	59	50.65%	31,410	1.3062	89.83%
2000	342	52.60%	42,980	1.8292	96.78%	86	56.29%	47,482	1.7223	94.19%
2001	591	50.99%	91,164	1.6025	93.06%	123	59.44%	104,766	1.3550	85.37%
2002	722	51.52%	169,484	1.1002	80.61%	101	50.33%	155,291	1.0493	79.21%
2003	341	30.65%	46,261	0.9396	82.99%	64	33.04%	40,040	0.9281	81.25%
2004	167	18.81%	19,579	0.6996	76.05%	37	23.43%	19,455	0.8233	81.08%
2005	74	30.28%	24,349	0.5904	62.16%	17	30.29%	24,826	0.8475	88.24%
2006	25	11.55%	2,088	0.3829	88.00%	5	-5.27%	1,433	0.3472	80.00%
Total	3,902	43.21%	555,777	1.3795	83.88%	871	46.88%	555,682	1.3633	83.58%

Table 2 - LGD By Cohort Year: Discussion

- ❑ Credit cycle reflected in peak count for 1991, but LGD is out of sync, peaking earlier in 1988 (1989) for instruments (obligors)

- ❑ 2nd downturn episode defaults peak in 2001 with *loca*/LGD peaks 2000 (2001) for instruments (obligors)
 - ❑ But an earlier peak for both in 1998

- ❑ The average dollar defaulted amount peaks near cyclical troughs in for instruments (obligors) 1991 (1990) and 2002

- ❑ Times-to-resolution seem to increase in the 1st but not 2nd recessionary period, with secular decline partly due to censoring

- ❑ Proportion of bankruptcy filings – a *weak* pattern of falling in the 2nd but rising in the 1st episode

Table 3 - LGD By Industry

Industry Group	Instruments			Obligors		
	Count	Average	Std. Dev.	Count	Average	Std. Dev.
Aerospace / Auto / Capital Goods / Equipment	350	35.36%	39.53%	90	41.92%	33.93%
Consumer / Service Sector	773	42.51%	41.56%	196	44.80%	30.28%
Energy / Natural Resources	462	36.43%	36.73%	84	39.72%	28.87%
Financial Institutions	124	40.48%	39.39%	28	54.45%	36.52%
Forest / Building Products / Homebuilders	171	38.03%	38.25%	33	45.98%	27.85%
Healthcare / Chemicals	261	42.86%	40.81%	67	49.44%	29.74%
High Technology / Telecommunications	852	56.60%	38.98%	144	60.38%	30.47%
Insurance and Real Estate	84	52.09%	37.40%	22	52.76%	31.16%
Leisure Time / Media	498	37.26%	40.24%	131	45.56%	29.46%
Transportation	244	45.56%	42.74%	63	38.93%	35.19%
Utilities	83	18.78%	24.49%	13	24.02%	40.04%
Grand Total	3,902	43.21%	40.37%	871	46.88%	31.79%

- ❑ 11 industry groups derived from the highest level NAIC segments
- ❑ Only 2 industries with significantly divergent average LGDs
- ❑ Utilities (24.0% and 18.8% at the obligor and instrument levels, respectively)
- ❑ High Technology / Telecommunications (60.4% / 56.6% at obligor / instrument levels, respectively)
- ❑ Consistent with the results of Altman and Kishore (1996)

Table 4 - LGD By Seniority and Collateral

		Revolving Credit / Term Loan		Senior Secured Bonds		Senior Unsecured Bonds		Senior Subordinated Bonds		Subordinated Bonds		Junior Subordinated Bonds		Other		Total Instrument	
Cash / Inventories / Receivables / Guarantee	Count / Average	125	1.15%	6	0.63%	0	N/A	0	N/A	1	96.98%	0	N/A	3	0.03%	135	1.81%
All or Non-Current Assets / Oil & Gas Reserves	Count / Average	650	18.66%	75	29.20%	0	N/A	1	42.51%	1	75.28%	0	N/A	1	-0.60%	728	19.83%
Most Assets / Real Estate	Count / Average	390	17.02%	146	28.91%	10	38.81%	0	N/A	1	96.98%	0	N/A	0	N/A	547	20.74%
Capital Stock / Inter-company Debt	Count / Average	126	27.37%	91	40.23%	0	N/A	2	84.05%	0	N/A	0	N/A	0	N/A	219	33.23%
2nd Lien	Count / Average	37	48.33%	126	64.80%	2	58.54%	8	26.57%	2	76.33%	0	N/A	0	N/A	175	59.63%
Plant, Property & Equipment	Count / Average	131	54.34%	30	49.86%	312	49.59%	84	72.54%	0	N/A	14	71.52%	0	N/A	571	54.61%
Intellectual Property	Count / Average	5	87.80%	4	46.13%	0	N/A	0	N/A	0	N/A	0	N/A	0	N/A	9	69.28%
Unsecured	Count / Average	82	36.28%	2	89.64%	596	55.03%	423	67.77%	369	67.22%	45	78.67%	1	98.58%	1518	61.30%
Total Collateral	Count / Average	1546	22.43%	480	41.87%	920	53.01%	518	67.92%	374	67.45%	59	76.97%	5	19.62%	3902	43.21%

- ❑ Unsecured instruments about 40% of instruments
- ❑ Most common security is All or Non-Current Assets / Oil & Gas Reserves
- ❑ LGD is much lower for secured (31.6%) than unsecured (61.3%)
- ❑ Cash security-lowest LGD (1.8%), Intellectual Property worst (69.3%)
- ❑ Loans-large part (37.3%), lowest LGD (20.3%) & mostly secured (92.0%)
- ❑ Average LGD is almost monotonically increasing in these creditor class categories

Table 6 – Instrument Level Regression Models

Variable	Model 1		Model 2		Model 3		Model 4		Model 5		Model 6	
	Partial Effect	P-Value	Partial Effect	P-Value	Partial Effect	P-Value	Partial Effect	P-Value	Partial Effect	P-Value	Partial Effect	P-Value
Debt to Equity Ratio (Market)	-0.0590	4.67E-10	-0.0434	4.99E-09	-0.1246	6.83E-10						
Book Value	0.0398	0.1423	0.0073	0.3868	0.0254	0.2318						
Tobin's Q	-0.0403	0.1210	-0.0539	0.0149	-0.0816	4.68E-03						
Working Capital / Total Assets	-0.1287	9.66E-04	-0.1109	5.59E-05	-0.1273	8.59E-04						
Operating Cash Flow	-0.0650	2.15E-05	-0.1036	2.40E-03	-0.1273	8.59E-04						
Profit Margin (Industry)	-0.0455	0.0363	-0.0960	4.55E-03	-0.0827	3.05E-03						
Senior Secured	-7.10E-03	0.4142	-1.60E-03	0.4719	-3.67E-03	0.4521	0.0371	5.85E-04	0.0232	0.0414	0.0530	1.69E-03
Senior Unsecured	0.0477	0.0386	0.0291	0.0545	0.0354	0.0579	0.0390	4.61E-03	0.0318	2.35E-03	0.0370	0.0102
Senior Subordinated	0.0675	0.0188	0.0417	0.0238	0.0640	0.0202	7.81E-03	0.0005	0.0343	2.24E-02	0.0643	2.89E-04
Junior Subordinated	0.1663	0.4463	0.0348	0.1695	0.1831	0.4488	0.0387	0.1385	-0.0156	0.3553	-0.0198	0.3254
Collateral Rank	0.1000	1.00E-10	0.0621	4.88E-14	0.0970	0.00E+00	0.0185	1.00E-02	0.0617	0.00E+00	0.0487	1.36E-11
Percent Debt Above	-0.1646	0.0560	-0.0508	2.40E-01	-0.1676	4.22E-02	-0.0639	4.78E-01	0.4436	0.00E+00	0.0658	1.41E-01
Percent Debt Below	-0.3665	0.0001	-0.2507	1.72E-04	-0.3637	9.77E-05	-3.96E-03	0.4778	-0.5137	0.00E+00	-0.0367	9.12E-10
Number of Creditor Classes	-0.0284	0.1771	-0.0130	0.2564	-2.07E-03	0.4707	-0.0646	0.0004	-0.0299	5.91E-03	-0.0218	0.1046
Percent Secured Debt	0.1009	0.1867	0.0429	0.2985	0.0738	0.2330	0.0749	0.0762	0.2140	1.41E-09	0.1000	0.0320
Percent Bank Debt	0.1817	0.0447	0.1631	0.0187	0.3332	1.61E-04	-0.1712	3.66E-04	0.1883	3.34E-09	-0.1691	1.68E-04
Percent Subordinated Debt	-0.1680	0.0543	-0.0831	0.0236	-0.0730	0.1175	-0.0796	0.0269	-0.0151	0.2843	-0.0938	0.0061
Investment Grade at Origination	-0.1214	3.75E-05	-0.1139	3.93E-06	-0.1671	1.47E-06	-0.0585	1.51E-03	-0.0797	1.36E-09	-0.1648	6.36E-11
Principal at Default	0.0826	1.71E-03	0.0554	9.23E-04	0.0659	6.35E-04	-4.15E-03	0.4061	0.0953	0.00E+00	0.0692	1.59E-06
Industry - Utility	-0.7735	0.4934	-0.0905	0.1574	-0.1557	0.2187	-0.1300	9.53E-04	-0.0430	0.0778	-0.1050	9.03E-04
Industry - Technology	0.0468	0.0154	0.0235	0.0440	0.0837	1.35E-04	0.0476	4.15E-05	-0.0281	0.0046	0.1145	0.00E+00
Cumulative Abnormal Returns			-0.2777	3.95E-16					-0.2406	1.92E-19		
LGD at Default - Obligor					0.1754	5.48E-11					0.1062	1.58E-14
LGD at Default - Instrument	0.1815	1.51E-13					0.1626	2.39E-17				
Prepackaged Bankruptcy	-5.43E-03	0.3860	-4.23E-03	0.3751	-0.0547	-6.33E-16	-4.96E-03	9.65E-16	-5.58E-03	0.2483	-0.1089	-9.38E-15
Bankruptcy Filing	0.2256	3.75E-13	0.1213	3.55E-09	0.2671	-5.23E-15	0.0975	2.92E-11	0.0448	4.48E-06	0.2450	-2.92E-15
1989-1991 Recession	0.0902	0.0120	0.0437	0.0247	0.0931	0.0005	0.0692	0.0000	8.26E-03	0.2115	0.0793	1.04E-07
2000-2002 Recession	0.1377	2.37E-06	0.0797	2.83E-05	0.1227	1.60E-05	0.0798	1.27E-06	-4.10E-03	3.73E-01	0.0973	2.12E-08
Moody's Speculative Default Rate	-0.0227	5.210E-04	-0.0174	9.170E-05	-0.0291	0.3099	-0.0243	0.2260	0.0268	0.1217	-0.0371	0.1245
S&P 500 Return	-0.0826	0.4743	-0.1193	0.0187	-0.1660	0.1791	0.5625	4.56E-07	0.1291	0.0465	0.3000	2.55E-03
	In-Smpl	Out-Smpl	In-Smpl	Out-Smpl	In-Smpl	Out-Smpl	In-Smpl	Out-Smpl	In-Smpl	Out-Smpl	In-Smpl	Out-Smpl
Log of Observations	393	79	699	134	793	159	833	167	1523	305	1793	359
Log-Likelihood	1.14E-10	1.90E-08	1.12E-13	2.18E-11	2.31E-08	2.88E-06	6.21E-10	5.14E-07	2.86E-13	2.59E-10	6.84E-08	5.05E-05
Pseudo R-Squared	0.7046	0.5458	0.7926	0.6332	0.6211	0.4927	0.5754	0.4360	0.4369	0.3575	0.5206	0.3931
Hosmer-Lemeshow	0.4105	0.2115	0.4755	0.2442	0.3671	0.1891	0.3116	0.1876	0.3739	0.2853	0.2640	0.1962
Area under ROC Curve	0.8104	0.7378	0.8876	0.7952	0.7275	0.6537	0.7074	0.6389	0.7385	0.6629	0.6758	0.6027
Kolmogorov-Smirnov	4.18E-06	4.01E-05	2.92E-07	2.31E-06	1.58E-05	8.67E-04	6.85E-05	2.20E-04	1.69E-06	2.56E-05	5.23E-03	0.0380

- We estimate BLGLM and BTLM models with and without financial ratios
- Different measures of equity or debt market information at default – trading price of debt (facility and obligor level) vs. cumulative abnormal stock returns
- The BLGLM models perform better – signs / significance levels or estimates and discrimination / predictive accuracy

Beta Link Generalized Linear Model

Beta Transformed Linear Model

Table 7 – Obligor Level Regression Models

Category	Variable	Model 1		Model 2		Model 3		Model 4	
		Partial Effect	P-Value	Partial Effect	P-Value	Partial Effect	P-Value	Partial Effect	P-Value
Financial	Debt to Equity Ratio (Market)	-0.0352	1.76E-03	-0.0359	1.09E-03				
	Book Value	0.0775	0.1456	-0.0098	0.4436				
	Tobin's Q	0.0497	1.65E-03	0.0147	3.81E-03				
	Current Ratio	0.0187	0.2657	-0.0045	0.4318				
	Cash Flow from Operations	-4.39E-04	0.0205	-5.81E-04	4.04E-03				
	Profit Margin (Industry)	-0.0328	1.93E-03	-0.0287	2.11E-03				
Time	Time Between Defaults	-0.3063	0.0270	-0.0381	0.3669	0.0420	0.1427	0.0769	0.0310
	Time Since Issue	0.0101	0.1501	0.0082	0.1913	-2.31E-03	0.3666	-3.86E-03	0.0310
	Time-to-Maturity	0.0150	0.0729	0.0185	0.0162	0.0231	1.30E-05	0.0281	3.24E-08
Capital Structure	Number of Creditor Classes	-0.0655	0.0193	-0.0393	0.0741	-0.0287	0.0420	-0.0165	0.0765
	Percent Secured Debt	-0.1136	0.0672	-0.2064	0.0330	-0.0390	0.0901	-0.0890	0.0649
	Percent Bank Debt	-0.2147	0.0689	-0.0818	0.0861	-0.2939	1.66E-05	-0.2201	4.39E-04
Credit Quality / Market	Percent Subordinated Debt	-0.1416	0.1508	-0.0665	0.1835	-0.0403	0.1957	-0.0259	0.2862
	Investment Grade at Origination	-0.0666	0.0511	-0.0718	0.0899	-0.0239	0.1183	-0.0965	2.29E-03
	Principal at Default	8.71E-03	4.25E-03	1.04E-03	0.0237	4.61E-04	0.0461	2.56E-04	0.0374
	Industry - Utility	-0.1528	0.0500	-0.1299	0.0710	-0.0901	0.0663	-0.1263	0.0743
	Industry - Technology	0.0240	0.0619	0.0520	0.0594	0.0483	0.0106	0.0651	9.44E-04
	Cumulative Abnormal Returns	-0.2238	1.43E-10			-0.1753	5.58E-11		
Lag	LGDA at Default - Obligor			0.1616	1.43E-08			0.1153	5.58E-09
	Prepackaged Bankruptcy	-0.0205	0.0586	-0.0532	0.0199	-0.0717	3.79E-06	-0.0828	4.70E-07
Macro	Bankruptcy Filing	0.1230	4.30E-03	0.1542	1.68E-05	0.0147	6.33E-12	0.0162	6.81E-13
	1989-1991 Recession	0.0624	0.0944	0.0521	0.0614	1.42E-03	0.1187	8.56E-03	0.0912
	2000-2002 Recession	0.0928	0.0210	0.0997	0.0145	0.0843	2.64E-04	0.1119	1.62E-05
	Moody's All-Corporate Default Rate	0.0641	1.60E-04	0.0413	0.0747	0.0145	0.0161	0.0172	0.0139
	S&P 500 Return	-0.0111	3.45E-03	-0.0172	2.60E-03	-0.0189	8.73E-03	-0.0208	8.22E-03
		In-Smpl	Out-Smpl	In-Smpl	Out-Smpl	In-Smpl	Out-Smpl	In-Smpl	Out-Smpl
Diagnostics	Number of Observations	148	30	185	37	343	69	424	85
	Log-Likelihood	5.34E-09	5.43E-07	1.72E-08	1.73E-06	1.88E-07	1.88E-05	1.14E-06	1.14E-04
	Pseudo R-Squared	0.5851	0.4461	0.5435	0.4133	0.4634	0.3485	0.3749	0.2869
	Hoshmer-Lemeshow	0.5655	0.4319	0.4306	0.3297	0.2116	0.1599	0.1742	0.1404
	Area under ROC Curve	0.8943	0.8033	0.8616	0.7691	0.7357	0.6603	0.6690	0.5942
	Kolmogorov-Smirnov	4.18E-07	1.41E-06	2.19E-06	1.61E-05	4.46E-05	3.34E-05	2.92E-05	2.03E-04
Financial	Debt to Equity Ratio (Market)	-0.0248	0.0390	-0.0227	0.0535				
	Book Value	0.0508	0.1875	0.0363	0.2670				
	Tobin's Q	0.0378	1.79E-03	0.0205	3.14E-03				
	Current Ratio	0.0508	0.1875	-1.46E-03	0.4754				
	Cash Flow from Operations	-1.76E-04	0.0482	-2.03E-04	0.0337				
	Profit Margin (Industry)	-0.0325	0.0148	-0.0235	0.0233				
Time	Time Between Defaults	-0.1496	0.0970	0.0169	0.4303	0.0416	0.1377	0.0717	0.0329
	Time Since Issue	0.0102	0.1229	2.68E-03	0.3821	-6.63E-03	0.1438	-8.09E-03	0.0915
	Time-to-Maturity	0.0117	0.0857	0.0144	0.0299	0.0182	4.83E-05	0.0239	2.39E-08
Capital Structure	Number of Creditor Classes	-0.0547	0.0167	-0.0337	0.0871	-0.0264	0.0355	-0.0192	0.0648
	Percent Secured Debt	-0.0509	0.0033	-0.1708	0.0489	-0.0403	0.0025	-0.0922	0.0048
	Percent Bank Debt	-0.1662	0.0998	-0.0587	3.04E-04	-0.2522	3.54E-05	-0.1806	1.59E-03
Credit Quality / Market	Percent Subordinated Debt	-0.1361	0.2140	-0.0552	0.2107	-0.0327	0.2174	-0.1806	0.1580
	Investment Grade at Origination	-0.0494	1.67E-03	-0.0570	1.37E-03	-0.0159	3.03E-03	-0.0820	2.94E-04
	Principal at Default	3.14E-03	0.0513	2.25E-03	0.0962	1.41E-04	0.0407	1.02E-04	0.0435
	Industry - Utility	-0.1185	3.48E-05	-0.1579	7.06E-04	-0.1309	9.92E-03	-0.1403	5.68E-03
	Industry - Technology	0.0304	0.0276	0.0510	0.0360	0.0361	0.0235	0.0556	1.32E-03
	Cumulative Abnormal Returns	-0.3131	1.48E-08			-0.2462	1.48E-09		
Lag	LGDA at Default - Obligor			0.2694	1.48E-07			0.1987	1.48E-08
	Prepackaged Bankruptcy	-0.0127	0.0301	-0.0483	0.0229	-0.0610	2.47E-05	-0.0745	1.92E-07
Macro	Bankruptcy Filing	0.0985	5.20E-03	0.1456	1.03E-06	0.1364	5.50E-14	0.1496	7.11E-13
	1989-1991 Recession	0.0405	0.0822	0.0429	0.0693	1.34E-03	0.1248	6.51E-03	0.0959
	2000-2002 Recession	0.0757	0.0277	0.0941	0.0112	0.0843	2.64E-04	0.0988	3.01E-05
	Moody's All-Corporate Default Rate	0.0475	0.0278	0.0277	0.0140	0.0162	0.0201	0.0158	0.0140
	S&P 500 Return	-0.0103	0.0970	-0.0033	0.0446	-0.0116	0.0873	-0.0162	0.0121
		In-Smpl	Out-Smpl	In-Smpl	Out-Smpl	In-Smpl	Out-Smpl	In-Smpl	Out-Smpl
Diagnostics	Number of Observations	148	44	185	56	343	103	424	127
	Log-Likelihood	5.35E-06	5.35E-03	1.72E-05	1.72E-02	3.13E-01	3.13E-01	9.80E-03	9.80E-01
	Pseudo R-Squared	0.4780	0.3267	0.4364	0.2977	0.4537	0.2596	0.3490	0.1747
	Hoshmer-Lemeshow	0.2272	0.0779	0.2281	0.1304	0.1927	0.0841	0.1634	0.0455
	Area under ROC Curve	0.8368	0.6708	0.7405	0.5931	0.6391	0.5763	0.5306	0.5030
	Kolmogorov-Smirnov	1.71E-04	4.67E-03	1.93E-03	0.0748	0.0114	0.7338	0.2510	0.2292

- Like in the instrument level regressions, we estimate BLGLM and BTLM models with and without financial ratios
- Different measures of equity or debt market information at default – trading price of debt rolled up to obligor level vs. cumulative abnormal stock returns
- As in the instrument level, the BLGLM models perform better – signs / significance levels or estimates and discrimination / predictive accuracy

Beta Link Generalized Linear Model

Beta Transformed Linear Model

Full-Information Maximum Likelihood (FIML) Simultaneous Equation Estimation

- ❑ Model the relationship between LGD at the firm and instrument level (recovery on latter as a “collar option” on former)
 - ❑ Goal: understand structural determinants & improve forecasts

- ❑ Specification process: balance between fit, AUROC, signs & significance of estimates, economic sense:
 - ❑ Financial statement & capital structure (contractual feature) variables should only appear in obligor (instrument) equation

 - ❑ CARs best belong in the obligor equation and ultimate obligor LGD in the ultimate instrument LGD equation (feedback effect)

 - ❑ Moody’s Speculative Default Rate, SP500 return, Investment Grade, Utility, Prepack & 1989-91 only in obligor

 - ❑ Time-Between-Defaults (Time-to-Maturity) only in obligor (instrument)

 - ❑ Principal, Technology, Bankruptcy and 2000-2002 recession in both

FIML Simultaneous Equation Estimation

- Table of Results (Estimation)

Table 8 - FIML Simultaneous Equation Regression Analysis of Discounted Instrument and Obligor LGD (S&P and Moody's Rated Defaults 1985-2006)

Category	Instrument		Obligor		
	Variable	Partial Effect	P-Value	Partial Effect	P-Value
Financial	Debt to Equity Ratio (Market)			-0.0903	2.55E-03
	Book Value			-0.0814	0.0174
	Tobin's Q			0.0729	8.73E-03
	Intangibles Ratio (Industry Adjusted)			0.0978	7.02E-03
	Working Capital / Total Assets			-0.1347	4.54E-03
	Operating Cash Flow			-8.31E-03	0.0193
	Profit Margin (Industry)			-0.0917	1.20E-03
Contractual	Senior Secured	0.0432	0.0482		
	Senior Unsecured	0.0725	3.11E-03		
	Senior Subordinated	0.2266	1.21E-03		
	Junior Subordinated	0.1088	0.0303		
	Collateral Rank	0.1504	4.26E-12		
	Percent Debt Above	0.1241	3.84E-03		
	Percent Debt Below	-0.2930	7.65E-06		
Time	Time Between Defaults			-0.1853	7.40E-04
	Time-to-Maturity	0.0255	8.40E-03		
Capital Structure	Number of Creditor Classes			-0.1012	1.89E-04
	Percent Secured Debt			-0.1403	7.56E-03
	Percent Bank Debt			-0.2382	7.45E-03
Credit Quality / Market	Investment Grade at Origination			-0.0720	4.81E-03
	Principal at Default	8.99E-03	1.14E-03	1.35E-02	3.02E-02
	Industry - Utility			-0.1506	8.18E-03
	Industry - Technology	0.0608	2.03E-03	0.0343	5.72E-03
	Cumulative Abnormal Returns			-0.2753	1.76E-04
	Ultimate LGD - Obligor	0.5643	7.82E-06		
	LGD at Default - Obligor			0.1906	4.05E-04
Legal	LGD at Default - Instrument	0.2146	1.18E-14		
	Prepackaged Bankruptcy			-0.0406	5.38E-03
	Bankruptcy Filing	0.1835	3.70E-13	0.1429	5.00E-03
Macro	1989-1991 Recession			0.0678	0.0474
	2000-2002 Recession	0.0930	1.09E-02	0.1074	0.0103
	Moody's Speculative Default Rate			0.0726	1.72E-04
	S&P 500 Return			-0.0139	2.88E-04
Diagnostics		In-Smpl	Out-Smpl	In-Smpl	Out-Smpl
	Number of Observations	515	94	515	94
	Log-Likelihood	1.72E-10	9.60E-08	1.72E-10	9.60E-08
	Pseudo R-Squared	0.6997	0.6119	0.5822	0.4744
	Hoshmer-Lemeshow	0.4853	0.3907	0.4741	0.3608
	Area under ROC Curve	0.9375	0.8458	0.9012	0.8459
	Kolmogorov-Smirnov	1.12E-07	4.89E-06	1.42E-07	6.87E-06

Estimates all highly statistically significant

Good accuracy & discrimination

Partial effects: economic significance & reasonable magnitudes

Results are qualitatively similar and generally more robust than single equation regressions

Most important determinants of the ultimate instrument LGD: instrument LGD at default & ultimate obligor LGD

Feedback between the 2 levels of LGD

Partial effects of 0.22 & 0.56, respectively

Model Performance Comparison I: Single Equation BLGLM vs. KLRE vs. BDKE

			Instrument			Obligor		
	Test Statistic	Model	Beta Link GLM	KLRE	Beta Density Kernel	Beta Link GLM	KLRE	Beta Density Kernel
Out-of-Sample / Time 1 Year Ahead Prediction	Area Under Receiver Operating Characteristic Curve ²	Median	0.7198	0.6858	0.6183	0.7682	0.6910	0.5484
		Standard Deviation	0.0995	0.1012	0.1059	0.0921	0.0952	0.0997
		5th Percentile	0.5206	0.4794	0.4061	0.5818	0.4921	0.3446
		95th Percentile	0.9095	0.8818	0.8286	0.9482	0.8807	0.7434
	Komogorov-Smirnov Statistic ³	Median	6.81E-05	7.16E-03	1.21E-01	7.15E-05	9.17E-03	8.00E-02
		Standard Deviation	4.26E-07	7.17E-05	1.21E-03	3.63E-07	9.18E-05	8.09E-04
		5th Percentile	2.90E-05	6.69E-03	9.62E-02	3.25E-05	8.70E-03	5.54E-02
		95th Percentile	6.79E-02	9.35E-02	3.71E-01	3.07E-02	7.95E-02	4.67E-01
	McFadden Pseudo R-Squared ⁴	Median	0.5442	0.4926	0.3933	0.5067	0.4617	0.3610
		Standard Deviation	0.0999	0.1329	0.1634	0.1062	0.1327	0.1643
		5th Percentile	0.3354	0.2234	0.0650	0.2885	0.1910	0.0300
		95th Percentile	0.7343	0.7496	0.7157	0.7179	0.7242	0.6827
	Hoshmer-Lemeshow Chi-Squared (P-Values) ⁵	Median	0.2318	0.1041	0.0537	0.3193	0.1321	0.0728
		Standard Deviation	0.0320	0.0306	0.0323	0.0386	0.0373	0.0312
		5th Percentile	0.1594	0.0405	0.0030	0.2320	0.0504	0.0043
		95th Percentile	0.2941	0.1626	0.1182	0.3958	0.2044	0.1303

- At both the instrument and obligor levels, in a resampled out-of-time and out-of-sample validation exercise, the “parameteric” BLGLM model outperforms the “semi-parametric” KLRE, which in turn outperforms the fully non-parametric BDKE (for both discriminatory and predictive accuracy)

Model Performance Comparison I: Single Equation BLGLM vs. KLRE vs. BDKE (cont'd)

Fig. 1 - Densities of AUROCs for Instrument LGD Prediction
100,000 Repetitions Out-of-Sample and Out-of-Time 1996-2006

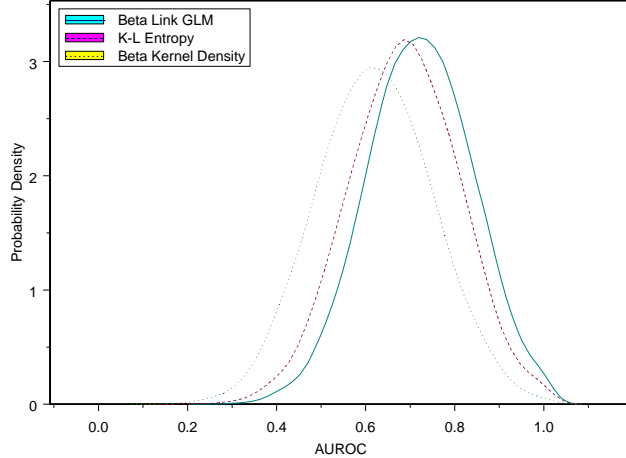


Fig.3 - Densities of KS P-Values for Instrument LGD Prediction
100,000 Repetitions Out-of-Sample and Out-of-Time 1996-2006

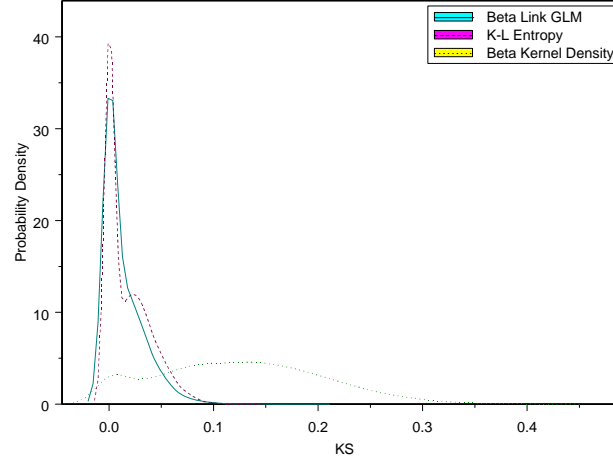


Fig. 2 - Densities of AUROCs for Obligor LGD Prediction
100,000 Repetitions Out-of-Sample and Out-of-Time 1996-2006

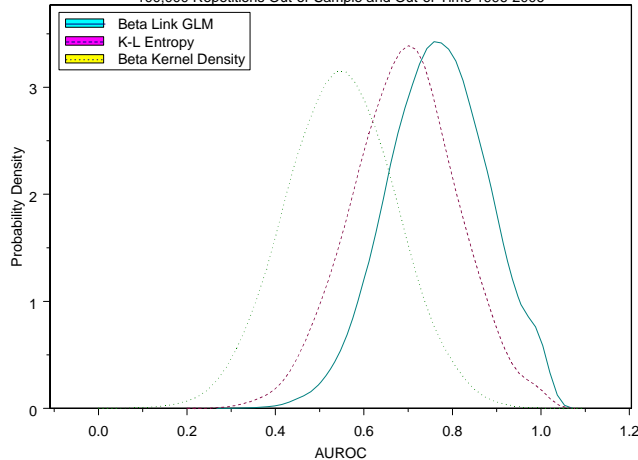
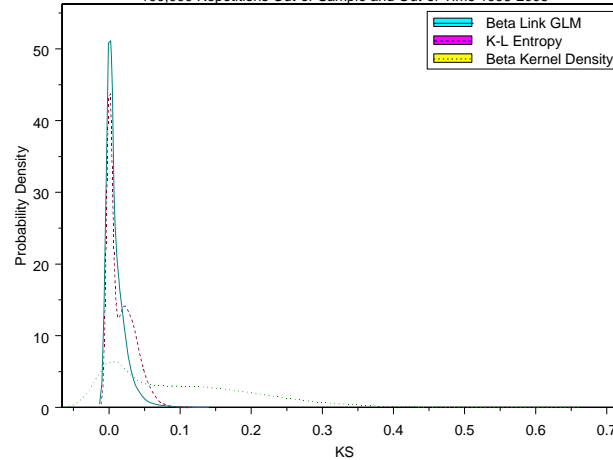


Fig.4 - Densities of KS P-Values for Obligor LGD Prediction
100,000 Repetitions Out-of-Sample and Out-of-Time 1996-2006



- Resampled densities of AUROCs shifted to the right for BLGLM vs. KLRE & BDKE, indicative of greater discrimination
- KS p-values concentrated near zero for BLGLM as compared to the 2 other models -> same conclusion

Model Performance Comparison I: Single Equation BLGLM vs. KLRE vs. BDKE (cont'd)

Fig. 5 - Densities of Pseudo-Rsquareds for Instrument LGD Prediction
100,000 Repetitions Out-of-Sample and Out-of-Time 1996-2006

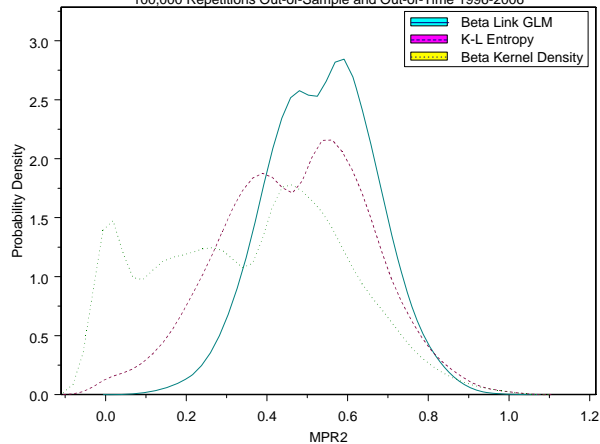


Fig.7-Densities of Hoshmer-Lemeshow P-Values for Instrument LGD
100,000 Repetitions Out-of-Sample and Out-of-Time 1996-2006

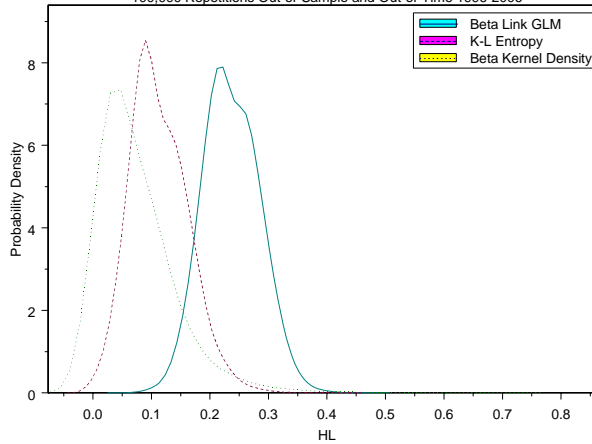


Fig. 6 - Densities of Pseudo-Rsquareds for Obligor LGD Prediction
100,000 Repetitions Out-of-Sample and Out-of-Time 1996-2006

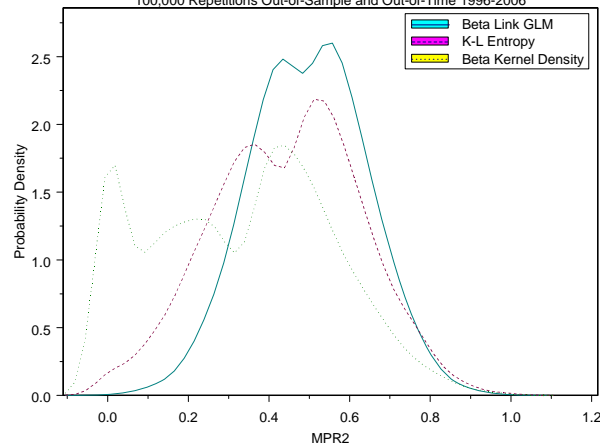
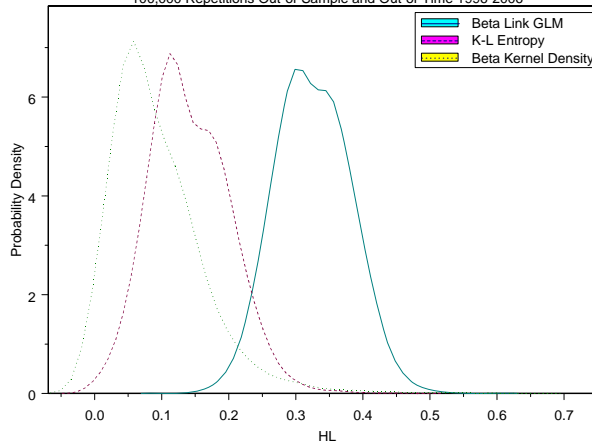


Fig.8-Densities of Hoshmer-Lemeshow P-Values for Obligor LGD Prediction
100,000 Repetitions Out-of-Sample and Out-of-Time 1996-2006



- Resampled densities of r-squareds shifted to the right for BLGLM vs. KLRE & BDKE -> more predictive accuracy
- HL p-values further from zero for BLGLM as compared to the 2 other models -> same conclusion

Model Performance Comparison II: BLGLM Single vs. FIML 2-Equation

			Instrument		Obligor	
		Model	Single Equation	Simulataneous Equation	Single Equation	Simulataneous Equation
Out-of-Sample / Time 1 Year Ahead Prediction	Area Under Receiver Operating Characteristic Curve ²	Median	0.7198	0.7910	0.7682	0.7916
		Standard Deviation	0.0995	0.1070	0.0921	0.1054
		5th Percentile	0.5206	0.5684	0.5818	0.5803
		95th Percentile	0.9095	0.9963	0.9482	0.9987
	Komogorov-Smirnov Statistic ³	Median	1.07E-04	1.12E-06	8.78E-05	3.47E-07
		Standard Deviation	1.15E-06	1.44E-08	1.16E-06	3.79E-08
		5th Percentile	9.03E-05	1.58E-06	1.51E-05	3.23E-07
		95th Percentile	3.62E-02	8.52E-03	9.64E-02	2.64E-02
	McFadden Pseudo R-Squared ⁴	Median	0.5442	0.6034	0.5067	0.5611
		Standard Deviation	0.0999	0.1243	0.1062	0.1322
		5th Percentile	0.3354	0.3476	0.2885	0.2869
		95th Percentile	0.7343	0.8485	0.7179	0.8234
	Hoshmer-Lemeshow Chi-Squared (P-Values) ⁵	Median	0.2318	0.4771	0.3193	0.4963
		Standard Deviation	0.0320	0.0428	0.0386	0.0438
		5th Percentile	0.1594	0.3858	0.2320	0.4076
		95th Percentile	0.2941	0.5547	0.3958	0.5784

- The simultaneous equation model outperforms the single equation models across all measures: distributions of AUROCs, MPR2 and HL (KS) shifted to the right (left), or better discriminatory power and predictive accuracy

Model Performance Comparison II: BLGLM Single vs. FIML 2-Equation (cont'd)

Fig. 9 - Densities of AUROCs for Instrument LGD Prediction
100,000 Repetitions Out-of-Sample and Out-of-Time 1996-2006

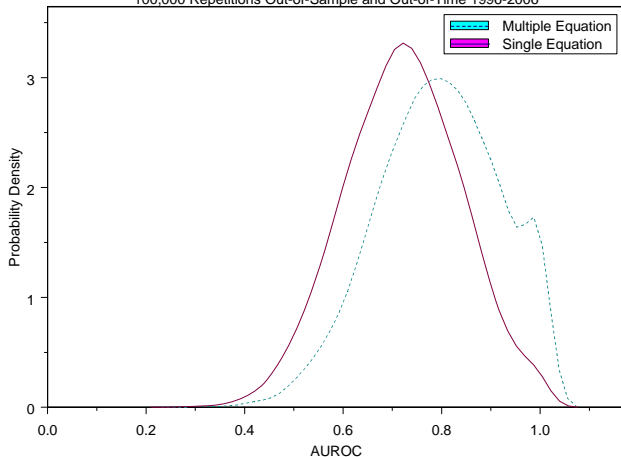


Fig.11 - Densities of KS P-Values for Instrument LGD Prediction
100,000 Repetitions Out-of-Sample and Out-of-Time 1996-2006

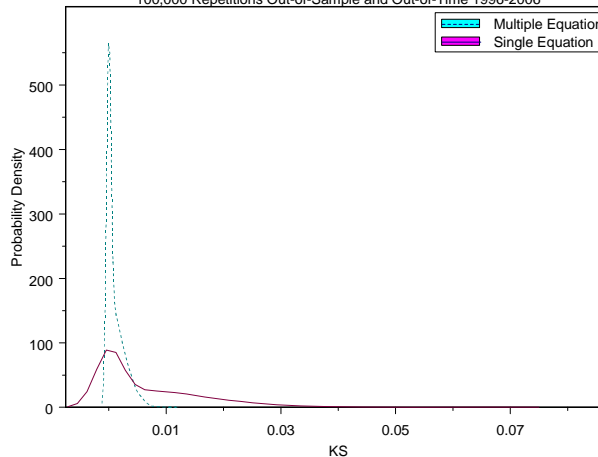


Fig. 10 - Densities of AUROCs for Obligor LGD Prediction
100,000 Repetitions Out-of-Sample and Out-of-Time 1996-2006

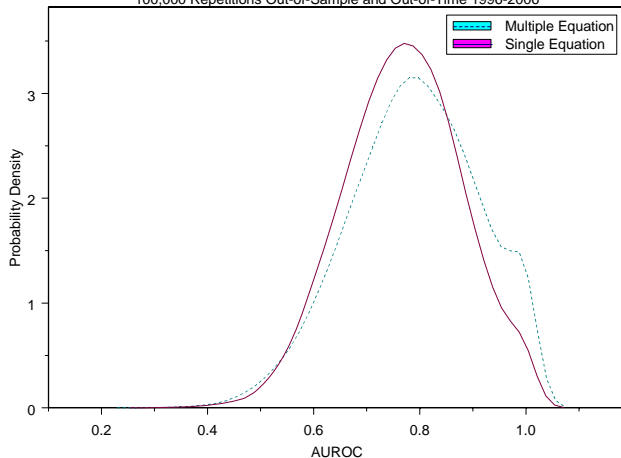
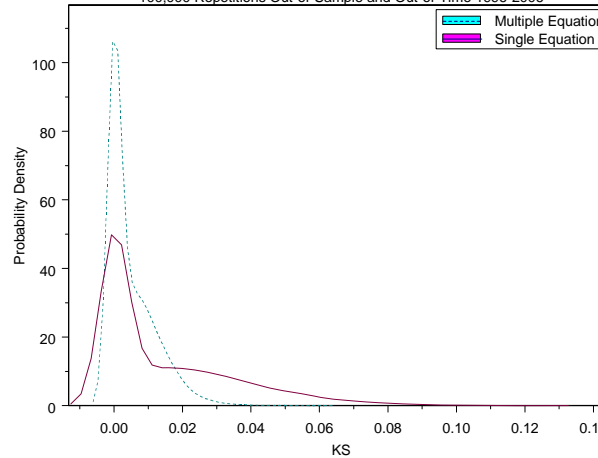


Fig.12 - Densities of KS P-Values for Obligor LGD Prediction
100,000 Repetitions Out-of-Sample and Out-of-Time 1996-2006



- Resampled densities of AUROCs shifted to the right for simultaneous vs. single equation models, indicative of greater discrimination
- KS p-values concentrated near zero for 2 vs. 1-equation models -> same conclusion

Model Performance Comparison II: BLGLM Single vs. FIML 2-Equation (cont'd)

Fig. 13 - Densities of Pseudo-Rsquareds for Instrument LGD Prediction
100,000 Repetitions Out-of-Sample and Out-of-Time 1996-2006

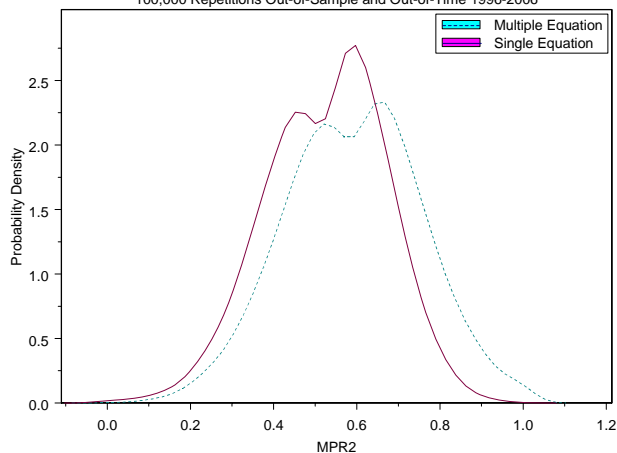


Fig.15-Densities of Hoshmer-Lemeshow P-Values for Instrument LGD
100,000 Repetitions Out-of-Sample and Out-of-Time 1996-2006

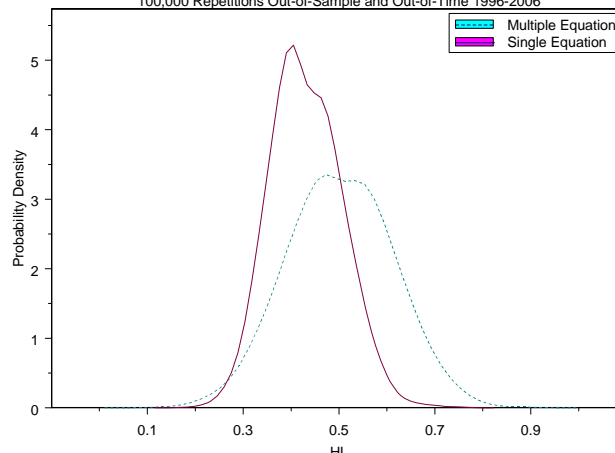


Fig. 14 - Densities of Pseudo-Rsquareds for Obligor LGD Prediction
100,000 Repetitions Out-of-Sample and Out-of-Time 1996-2006

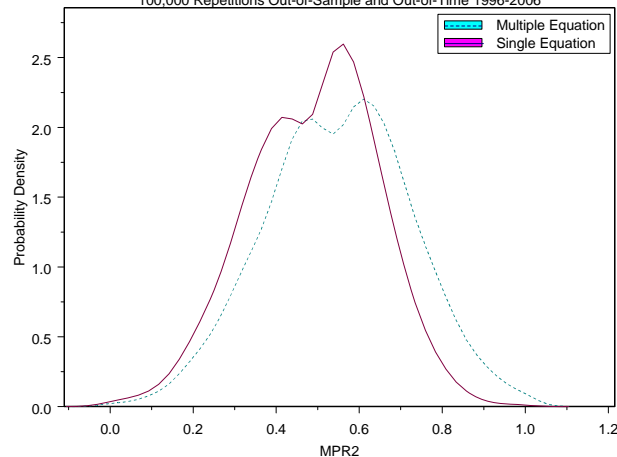
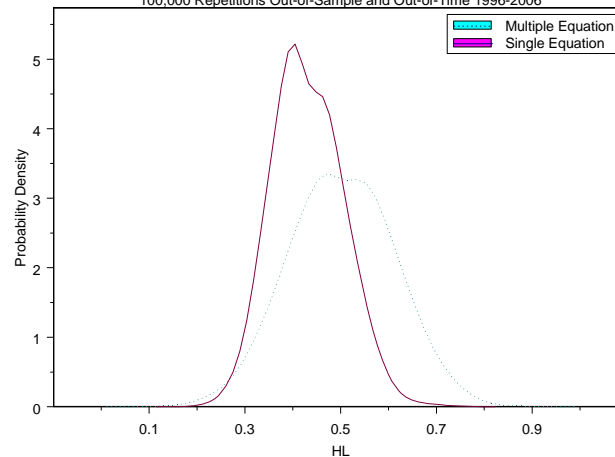


Fig.16-Densities of Hoshmer-Lemeshow P-Values for Obligor LGD
100,000 Repetitions Out-of-Sample and Out-of-Time 1996-2006



- Resampled densities of r-squareds shifted to the right for single vs. multiple equation models -> more predictive accuracy
- HL p-values further from zero for 1- vs. 2-equation r models -> same conclusion

Model Performance Comparison II: BLGLM Single vs. FIML 2-Equation (cont'd)

- ❑ AUROC rank ordering increases: medians in the instrument (obligor) 0.72 to 0.79 (0.77 to 0.79) out-of-sample
 - ❑ But standard deviation increases slightly 0.099 to 0.107 (0.092 to 0.105)
- ❑ KS P-Values distributions median out-of-sample decline from $0.07e^{-4}$ to $1.12e^{-6}$ ($8.78e^{-5}$ to $3.47e^{-7}$) in the instrument (obligor) equation
 - ❑ And standard deviation decreases 1.15×10^{-6} to 1.44×10^{-8} (1.16×10^{-6} to 3.79×10^{-8}) in the instrument (obligor) equation
- ❑ MPR2s better out-of-sample 0.54 to 0.60 (0.51 to 0.56) for instrument (obligor)
 - ❑ Decrease non-trivial manner increased dispersion (obligor/instrument from 0.099 to 0.12 / 0.11 to 0.13)
 - ❑ Note high degree of multimodality
- ❑ P-Values of HL statistics out-of-sample dramatic improvement
 - ❑ Median instrument (obligor) 0.23 to 0.48 (0.32 to 0.50)
 - ❑ Only slight increase in dispersion instrument (obligor) 0.032 to 0.043 (0.039 to 0.043)

Summary and Directions for Future Research

- ❑ Empirically investigate ultimate LGD at instrument & obligor levels with comprehensive data-set & rigorous econometric methodology
- ❑ Compare econometric models in different classes by rank ordering and predictive accuracy properties
- ❑ Show that a simultaneous equation version of a beta link GLM, has some desirable properties relative to single equation approaches
- ❑ Model is validated rigorously through a resampling experiment in a rolling out-of-time and out-of-sample framework
- ❑ Confirm previous findings & in addition significance of macro factor, equity returns, price of debt at default and obligor financials
- ❑ Extensions: theoretical models, Bayesianism, alternative data-sets