A Bivariate Generalized Autoregressive Conditional Heteroscedasticity-in-Mean Study of the Relationship between Return Variability and Trading Volume in International Futures Markets

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INTRODUCTION

The relationship between trading volume and price changes in futures markets continues to be of interest mainly due to the inconclusive nature of the results reported so far in the literature (Karpoff, 1987). One source of controversy centers on the empirical distribution of futures price changes (Sterge, 1989).[1]

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[1]This issue is less contentious in equity markets, where the lognormal distribution has been found to be a reasonable approximation, at least relative to speculative markets (Mandelbrot, 1963).

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A recent study by Najand and Yung (1991) concludes that price changes in U.S. Treasury bond futures markets are best characterized by a time series model that allows for generalized autocorrelation as well as conditional heteroscedasticity (GARCH) in the second moments. Using this GARCH framework, Najand and Yung (1991) document a positive relationship between price variability and trading volume, consistent with both the mixture of distributions hypothesis (Clark, 1973), several sequential equilibrium models of speculative markets (Copeland, 1976), and certain newer classes of noisy rational expectations equilibria (Blume, Easley, and O'Hara, 1994; Easley, Keifer, and O'Hara, 1994).

This paper extends Najand and Yung's (1991) study in two ways. First, a cross-section of international futures markets is examined. A study of international futures markets is important because it provides independent verification of the results obtained in domestic markets. Second, a bivariate exponential GARCH(1,1)-in-mean model is used. In addition to allowing for autocorrelation in the first and second moments, this model has the advantages of avoiding simultaneity bias with regard to the effect of volume on price volatility, allowing for nonlinearities in the second moments, as well as providing a means for estimating a risk premium.

This study is relevant to finance for at least three reasons. First, such a study yields insights into how information is disseminated in speculative markets and the degree to which it is conveyed by prices. This knowledge can be used to augment statistical power in tests of finance hypotheses. The second area to which this study has relevance is the ongoing debate about the distribution of speculative prices, which involves the competing stable Paretian and normal-lognormal mixture hypotheses, the latter deriving much of its empirical content from price-volume studies such as this one. Finally, it may be argued that this type of research has particular relevance to the theory and practice of futures markets. In particular, to the controversy over the effects of speculative activity in the creation and expansion of futures markets upon price volatility.[2] To the extent that volume can serve as a proxy for an information flow variable, it may be possible to improve the assessment of these effects, if better methods of discerning this price-volume relationship can be implemented. Therefore, the results of this paper will be relevant to technical analysis if trading volume is found to play a role in providing information on the quality of information contained in price statistics (LeBaron, 1992).

The results of this paper indicate that there is a positive relationship between trading volume and price volatility, as measured by the condi-

[2]The treatment of these issues goes back to Working (1953, 1963).

tional heteroscedasticity of price change, in international financial futures markets. Additionally, the paper documents new statistically significant findings of positive contemporaneous and time varying correlation between price changes and volume, negative time varying risk premia in futures return, and a monotonically declining and asymmetric effect of innovations on price volatility.

REVIEW OF THE LITERATURE

A positive relationship between trading volume and returns (or price changes) is relatively well established for equities. The seminal work in this area is Ying (1966), who applied cross-spectral methods to equity prices and trading volume of the S&P 500 index and New York Stock Exchange (NYSE), respectively. He found a positive correlation between volume on the one hand and both price changes and magnitudes on the other. Harris (1983) studied 479 individual equities at daily intervals and found that squared price changes generally vary directly with trading volume (although there was some nontrivial cross-security variation). Cornell (1981) analyzed 17 commodity futures markets and found a positive correlation between *changes* in both the average trading volume and the standard deviation of log-relatives at two-month intervals. This was extended to the then new market for futures on Treasury bills by Tauchen and Pitts (1983), who found a similar relationship. Grammatikos and Saunders (1986), in their study of five foreign currency contracts at daily intervals with Granger-causality tests, documented significantly positive results that reveal no maturity effect on price variability. Lamoureux and Lastrapes (1990) applied Bollerslev's (1986) GARCH methodology to test for both second-order dependence and volume-price variability relationships in spot equity markets. They found that the GARCH effects disappeared after volume was included in the model. Najand and Yung (1991) applied univariate GARCH methodology, with volume as an explanatory variable in the conditional variance, for futures on U.S. Treasury bonds. They found both significant GARCH and volume effects in the second moments of futures returns. The authors documented a positive volume-variability relationship, using only lagged volume in the conditional variance due to the problem of simultaneity bias.

ECONOMETRIC METHODOLOGY

This study employs a bivariate GARCH(1,1) extension of Bollerslev's (1986) methodology. The equations describing the model are given as follows:

$$\Delta f_t = \alpha_0 + \alpha_1 \sqrt{h_t^f} + u_t^f \tag{1}$$

$$v_t = \beta_0 + \beta_1 v_{t-1} + \beta_2 u_{t-1}^v + \beta_3 t + \beta_4 \sqrt{h_t^v} + u_t^v \tag{2}$$

$$\begin{pmatrix} u_t^f \\ u_t^v \end{pmatrix} \sim N(0, H_t) \tag{3}$$

$$H_t = \begin{pmatrix} h_t^f & |h_t^{fv}| \\ |h_t^{fv}| & h_t^v \end{pmatrix} \tag{4}$$

$$\begin{aligned}
\begin{pmatrix} \ln(h_t^f) \\ \ln(h_t^v) \\ \ln(|h_t^{fv}|) \end{pmatrix} &= \begin{pmatrix} \gamma_0 \\ \delta_0 \\ \ln(|\varepsilon_0|) \end{pmatrix} + \begin{pmatrix} \gamma_1 & 0 & 0 \\ 0 & \delta_1 & 0 \\ 0 & 0 & \varepsilon_1 \end{pmatrix} \begin{pmatrix} \ln(h_{t-1}^f) \\ \ln(h_{t-1}^v) \\ \ln(|h_{t-1}^{fv}|) \end{pmatrix} \\
&+ \begin{pmatrix} \gamma_2 & 0 & 0 \\ 0 & \delta_2 & 0 \\ 0 & 0 & \varepsilon_2 \end{pmatrix} \begin{pmatrix} u_{t-1}^f \\ u_{t-1}^v \\ \operatorname{sign}(u_{t-1}^f\, u_{t-1}^v)\sqrt{|u_{t-1}^f u_{t-1}^v|} \end{pmatrix} + \begin{pmatrix} \gamma_3 & 0 & 0 \\ 0 & \delta_3 & 0 \\ 0 & 0 & \delta_3 \end{pmatrix} \\
&\begin{pmatrix} |u_{t-1}^f| \\ |u_{t-1}^v| \\ \sqrt{|u_{t-1}^f u_{t-1}^v|} \end{pmatrix} + \begin{pmatrix} \gamma_4 & 0 & 0 \\ 0 & \delta_4 & 0 \\ 0 & 0 & \delta_4 \end{pmatrix} \begin{pmatrix} v_{t-1} \\ \Delta f_{t-1} \\ \operatorname{sign}(\Delta f_{t-1}\, v_{t-1})\sqrt{|\Delta f_{t-1} v_{t-1}|} \end{pmatrix}
\end{aligned} \tag{5}$$

$$L(\Theta)|Y,u = -\frac{1}{2}\sum_{t=0}^{T} (\ln(2\pi) + \ln|H_t| + u_t^T H_t^{-1} u_t) \tag{6}$$

In this notation, $f_t = \ln(F_t)$ is the natural logarithm of the near contract's closing futures price F_t; $\Delta f_t = f_t - f_{t-1}$ is the price log-relative; $v_t = \ln(V_t)$ is the natural logarithm of the level of trading volume, V_t; and $u_t = (u_t^f, u_t^v)^T$ is the vector of random disturbance terms for log-relative price and log volume at time, t, respectively, with zero mean vector, 0, and conditional variance-covariance matrix, H_t, with elements, vech(H_t) $= (h_t^f, |h_t^{fv}|, h_t^v)^T$, as the respective conditional variances and covariance. Y,u are time series of observations and disturbances, respectively, and $L(.|.)$ is the log-likelihood of the parameter vector, θ, conditional on the observations.

Equation (1) models the futures return as having a deterministic component, $\alpha_0 + \alpha_1\sqrt{h_t^f}$, the expected rate of price change given the information set at time, t, and a stochastic component, u_t^f, which is conditionally heteroscedastic and correlated with volume. α_0 is the unconditional expected rate of price change, and following Domowitz and Hakkio (1985) and Engle, Lilien, and Robbins (1987), the risk premium component, $\alpha_1\sqrt{h_t^f}$, is modeled as being proportional to the conditional heteroscedasticity of the futures return process. This is a proxy for the systematic risk associated with unanticipated movements in interest rates. Greater systematic risk associated with unanticipated shifts in the yield curve is reflected in innovations to the futures price change process, which, in turn, directly influences conditional variance in the GARCH equation. One can say that the conditional heteroscedasticity proxies for systematic risk, and it is expected that the estimated coefficient, α_1, would be negative.[3] The law of motion for the logarithm of volume, eq. (2), has deterministic and stochastic components as well. The normal volume component is modeled as an ARMA(1,1) process[4] with a time trend.[5] The conditional standard deviation, $\sqrt{h_t^v}$, is included as a proxy for an informationally driven component. This is because trading activity is, by hypothesis, related to the level of volatility. The innovation, u_t^v, is interpreted as abnormal volume. In many asymmetric information models of trading volume, it is expected that there is some persistence in abnormal volume following an information event (Karpoff, 1986). The use of the conditional volatility in volume allows one to separate surges in volume due to informed market participants from those due to the uninformed as well as from surprises. To the extent that new information arrival associated with increased asymmetry of information among traders results in an increase in trading volume (Karpoff, 1987), and may be proxied for by $\sqrt{h_t^v}$, the estimated coefficient, β_4, is expected to be positive. Also, the persistence of abnormal volume might have implications for the theoretical properties of technical trading rules. According to Blume et al. (1994), volume is informative about the distribution of the price signal received by traders, so that one expects surges in volume due to informed trade to exhibit a degree of persistence, which is captured by the econometric specification. In this regard, the autocorrelation coefficient $\beta_1 > 0$, and the moving average coefficient $\beta_2 < 0$, as informed surges in volume persist and noisy innovations are subsequently discounted.

Equation (5) describes a bivariate exponential GARCH(1,1) structure for the second moments.[6] The cross-equation restrictions constrain the conditional moments to depend only upon their past levels, mean

[3]A theoretical rationale for this specification can be found in Engle et al. (1987).

[4]See Weiss (1984) for combining the Box-Jenkins style ARMA and GARCH time series model.

[5]In exploratory regressions volume is found to have a highly significant, positive time trend, similar to that found in Grammatikos and Saunders (1986). This can be attributed to growth in the cases of the Sterling, Bund, and European Currency Unit (ECU) markets. In the cases of Treasury bond and Eurodollar, an explanation for the relatively weaker relationships is that the contracts were delisted from London International Financial Futures Exchange (LIFFE) on June 1993 and March 1996, respectively.

[6]Vector Akaike and Bayesian information criteria analyses (available upon request), as well as the interests of parsimony and computational ease, motivate this lag-length specification and functional form.

equation innovations, and lagged levels of the *other* variable.[7] Estimation focuses on four key terms. First, log-volume in the conditional variance of the futures return models the effect of information flow upon price change through the volatility of return, which is in traders' information sets and, as such, is separate from the contemporaneous correlation of the innovations. Consistent with both the mixture of distributions hypothesis (MDH) and several models of sequential information transmission and noisy rational expectations equilibrium, the coefficient, γ_4, is predicted to have a positive sign. Second, the lagged return in the conditional variance of volume models the informational impact of price on volume. To the extent that price increases signal lower systematic risk, so that there is less hedging and/or speculative activity relative to informationally motivated trade, the expectation is that the coefficient estimate of δ_4 will be positive. Third, the contemporaneous correlation between price change and volume is measured by the coefficient, ε_0. The MDH, sequential information, and noisy rational expectations models suggest that this coefficient should be positive. Although this coefficient is estimated through a log-function, therefore implicitly restricted to be positive, a justification is that the weight of both the empirical and theoretical literature favors a non-negative correlation. This study is constrained by the requirements of the exponential class of GARCH econometric models.[8] Finally, any asymmetries in the response of the conditional return variance to innovations of differing signs are captured by including their absolute values as separate arguments to the conditional variance. Unfortunately, here there is no clear theoretical guidance. Nelson (1981) found, for spot Treasury bills, that volatility increases more for negative innovations than for positive innovations of equal magnitude.[9] If it were the case that $\gamma_2 > 0$, $\gamma_3 < 0$, and $|\gamma_3| < |\gamma_2|$, then this pattern would obtain.

Asymptotically efficient estimators of these parameters are obtained by the exact maximum likelihood method, which needs only the specification of some arbitrary initial conditions to perform the maximization. If, in addition to the distributional assumption, the standard regularity

[7] Including contemporaneous variables results in difficulty of interpretation, more complex asymptotics, and less tractable estimation (Hamilton, 1994).

[8] A constant correlation model with no sign restriction on this coefficient, in style similar to Bollerslev (1990), was estimated as a check. The results, available upon request, were not qualitatively different. Tests showed the correlation to exhibit time variation, so that the present model better captures the dynamics of the data.

[9] The leverage effect has been proposed as a reason for this phenomenon (see Black, 1976; Christie, 1982) in the context of equities. Negative price innovations lower the value of equity, increasing the debt-equity ratio of the firm, and therefore increase the riskiness of its cash flows as measured by the conditional volatility. It is not clear if this is applicable to interest rate futures.

conditions hold, then these estimators are also asymptotically normal, and the classical inference procedures are valid (Hamilton, 1994).[10] The log-likelihood for this model is given by eq. (6). The convergence algorithm employed is the method of Berndt, Hall, Hall, and Hausman (1974), which relies on the gradient vector to compute the asymptotic variance-covariance matrix.

DATA DESCRIPTION

The data set consists of daily observations on opening price, high and low price, closing price, trading volume, and open interest for six futures contracts, all traded for three-month cycles, on the London International Financial Futures Exchange (LIFFE). The underlying are all interest rate assets representing investments in various international money and bond markets: Sterling, Eurodollar, U.S. Treasury bond, German Government bond (Bund), 3-month European Currency Unit (ECU), and the Euromark. The beginning trading days on the contracts are 11/5/82, 10/1/82, 10/27/84, 9/29/88, 10/27/89, and 4/21/89, respectively. All of the series end on the trading day 6/27/94, except U.S. Treasury bond ending 6/27/93,[11] to yield series lengths of 2935, 2967, 2104, 1452, 1178, and 1302, respectively. The time series are formed by using the nearest-to-expire contract on each trading day, except in the month of expiration of the contract, where closing prices of the next near contract are used. The price change variable is calculated as the log-relative of the daily closing price, which is matched with the logarithm of trading volume for the trading day.

ESTIMATION RESULTS

Distributional and diagnostic statistics are presented in Tables I and II for the log-relatives of the near contract closing price and the logarithm of trading volume, respectively. In Table I, various measures show that all series exhibit substantial deviations from normality. There is statistically significant positive skewness in five of six cases. Kurtosis is far in excess of three in all cases. Consequently, the Berra-Jarque *J*-statistics for

[10]If conditional normality fails to hold, but the first two conditional moments are correctly specified, it can be shown that the quasimaximum likelihood estimators that obtain will still be consistent and asymptotically normal, under suitable technical conditions and an adjustment of the standard errors (Bollerslev and Wooldridge, 1989). The ROBUSTERRORS option in RATS is employed to account for the latter.

[11]The Treasury bond contract was delisted from LIFFE that month. Subsequent to the sample period, in March 1996, the Eurodollar was delisted as well.

TABLE I

Distributional Statistics and Diagnostic Tests on International Financial Futures Daily Closing Log Price Change Series of Near-Month Contract

Statistic	*Sterling*	*Eurodollar*	*U.S. Treasury Bond*	*Bund*	*ECU*	*Euromark*
Sample size (period)	2935 (11/5/82–6/27/94)	2967 (10/1/82–6/27/94)	2104 (10/27/84–6/27/93)	1452 (9/29/88–6/27/94)	1178 (10/27/89–6/27/94)	1302 (4/21/89–6/27/94)
Sample mean (μ)	−0.00474 (0.092)[a]	0.00619 (0.0011)[c]	0.01768 (0.2596)	−.00167 (0.0091)[c]	−.00346 (0.1685)	−.0017 (0.4847)
SD (σ)	0.15259	0.10340	0.71919	0.34731	0.0863	0.08824
Skewness (s)	2.2424 (0.00)[c]	2.21426 (0.00)[c]	0.61465 (0.00)[c]	−0.18278 (0.1340)	0.8293 (0.00)[c]	−4.15230 (0.00)[c]
Kurtosis (κ)	59.7558 (0.00)[c]	33.9556 (0.00)[c]	9.30827 (0.00)[c]	5.4495 (0.00)[c]	9.4661 (0.00)[c]	45.3231 (0.00)[c]
Berra-Jarque (J)-statistic	4.39×10^5 (0.00)[c]	715.72 (0.00)[c]	7728.3 (0.00)[c]	1798.95 (0.00)[c]	4533.2 (0.00)[c]	1.15×10^5 (0.00)[c]
Kolmogorov-Smirnov (D)-statistic[d]	0.1186 (0.036)[c]	0.0785 (0.0509)[c]	0.0136 (0.0270)	0.1282 (0.0509)[c]	0.1358 (0.0509)[c]	0.0161 (0.0381)
Ljung-Box (Q_{LB})-statistic	152.94 (0.00)[c]	147.22 (0.00)[c]	79.548 (0.00)[c]	98.6864 (0.00)[c]	119.2056 (0.00)[c]	99.4346 (0.00)[c]
Granger-Haugh (Q_{GH})-statistic	82.0503 (0.00)[c]	54.1601 (0.0817)[a]	24.2852 (0.9823)	73.0886 (0.002)[c]	37.3692 (0.6328)	229.48 (0.00)[c]
Dickey-Fuller (Z_{DF})-statistic	−3454.93 (0.00)[c]	−3306.70 (0.00)[c]	−2110.92 (0.00)[c]	−1717.09 (0.00)[c]	−1404.56 (0.00)[c]	−1300.54 (0.00)[c]
Phillips-Perron (Z_{PP})-statistic	−3704.48 (0.00)[c]	−3704.48 (0.00)[c]	−3704.48 (0.00)[c]	−3704.48 (0.00)[c]	−3704.48 (0.00)[c]	−3704.48 (0.00)[c]
Engle $T \times R^2$-statistic	107.13 (0.00)[c]	194.79 (0.00)[c]	225.41 (0.00)[c]	279.92 (0.00)[c]	296.84 (0.00)[c]	27.679 (0.00)[c]

Notes: The Berra-Jarque J-statistic, computed from the sample kurtosis and skewness, is distributed, $\chi^2(2)$, under the null hypothesis of normality. The Kolmogorov-Smirnov D-statistic is computed from the sample cumulative distribution of the prewhitened series and is distributed around zero under normality. The Ljung-Box Q_{LB}-statistic has an asymptotic $\chi^2(64)$ distribution under the null hypothesis of uncorrelated increments, where 64 is the number of squared autocorrelations in the prewhitened series. The Granger-Haugh Q_{GH}-statistic is computed from the cross-correlations of the price change and volume series at ± 20 lags and has an asymptotic $\chi^2(40)$ distribution under the null hypothesis of no Granger feedback between the series. The Dickey-Fuller Z_{DF}-statistic is computed from the first-order correlation coefficient of the prewhitened series and is distributed around zero under the null hypothesis of a unit root in the series. The Phillips-Perron Z_{PP}-statistic is a stationarity test that is robust to heteroscedasticity. Engle's $T \times R^2$-statistic from VAR(5) residuals has an asymptotic $\chi^2(5)$ distribution under H_0 of no second-order dependence. The critical values for the Ljung-Box/Granger-Haugh/Engle $T \times R^2$ test are 78.9/51.9/9.3, 83.7/55.8/11.2, and 93.3/63.3/15.2 computed from the CDF of the $\chi^2(64)/\chi^2(40)/\chi^2(5)$ distribution, respectively. The critical values for both the Dickey-Fuller and Phillips-Perron test are −5.7, −8.1, and −13.8.

[a–c]Significant at the 10%, 5%, and 1% levels, respectively.

[d]Approximate critical values at 18 maximum gap frequency for D.

TABLE II

Distributional Statistics and Diagnostic Tests on International Financial Futures Daily Log Trading Volume Series of Near-Month Contract

Statistic	*Sterling*	*Eurodollar*	*U.S. Treasury Bond*	*Bund*	*ECU*	*Euromark*
Sample size (period)	2935 (11/5/82–6/27/94)	2967 (10/1/82–6/27/94)	2104 (10/27/84–6/27/93)	1452 (9/29/88–6/27/94)	1178 (10/27/89–6/27/94)	1302 (4/21/89–6/27/94)
Sample mean (μ)	111.07 (0.00)[c]	26.041 (0.00)[c]	37.936 (0.00)[c]	510.29 (0.00)[c]	7.2995 (0.00)[c]	169.51 (0.00)[c]
SD (σ)	124.829	22.497	34.6982	449.901	7.9822	179.79
Skewness (s)	1.8601 (0.00)[c]	1.8601 (0.00)[c]	2.4186 (0.00)[c]	2.0172 (0.1340)	2.1573 (0.00)[c]	1.98227 (0.00)[c]
Kurtosis (κ)	48.2089 (0.00)[c]	11.0511 (0.00)[c]	14.8757 (0.00)[c]	4.8705 (0.00)[c]	6.4692 (0.00)[c]	5.5028 (0.00)[c]
Berra-Jarque (J)-statistic	4639.2 (0.00)[c]	1124.3 (0.00)[c]	2.15×10^4 (0.00)[c]	2419.9 (0.00)[c]	2967.89 (0.00)[c]	2495.4 (0.00)[c]
Kolmogorov-Smirnov (D)-statistic[d]	0.0498 (0.036)[c]	0.0280 (0.0381)	0.0348 (0.0301)[b]	0.0261 (0.0381)	0.0367 (0.0381)	0.0345 (0.0381)
Ljung-Box (Q_{LB})-statistic	181.66 (0.00)[c]	161.73 (0.00)[c]	201.91 (0.00)[c]	194.15 (0.00)[c]	205.03 (0.00)[a]	212.34 (0.00)[b]
Granger-Haugh (Q_{GH})-statistic	82.0503 (0.00)[c]	54.1601 (0.0817)[a]	24.2852 (0.9823)	73.0886 (0.002)[c]	37.3692 (0.6328)	229.48 (0.00)[c]
Dickey-Fuller (Z_{DF})-statistic	−2868.61 (0.00)[c]	−2917.83 (0.00)[c]	−2057.97 (0.00)[c]	−1450.90 (0.00)[c]	−1177.71 (0.00)[c]	−1302.12 (0.00)[c]
Phillips-Perron (Z_{PP})-statistic	−2760.79 (0.00)[c]	−2846.52 (0.00)[c]	−2071.99 (0.00)[c]	−1440.80 (0.00)[c]	−1159.50 (0.00)[c]	1305.77 (0.00)[c]
Engle $T \times R^2$-statistic	384.50 (0.00)[c]	147.04 (0.00)[c]	67.890 (0.00)[c]	365.275 (0.00)[c]	233.01 (0.00)[c]	124.58 (0.00)[c]

Notes: The Berra-Jarque J-statistic, computed from the sample kurtosis and skewness, is distributed, $\chi^2(2)$, under the null hypothesis of normality. The Kolmogorov-Smirnov D-statistic is computed from the sample cumulative distribution of the prewhitened series and is distributed around zero under normality. The Ljung-Box Q_{LB}-statistic has an asymptotic $\chi^2(64)$ distribution under the null hypothesis of uncorrelated increments, where 64 is the number of squared autocorrelations in the prewhitened series. The Granger-Haugh Q_{GH}-statistic is computed from the cross-correlations of the price change and volume series at ± 20 lags and has an asymptotic $\chi^2(40)$ distribution under the null hypothesis of no Granger feedback between the series. The Dickey-Fuller Z_{DF}-statistic is computed from the first-order correlation coefficient of the prewhitened series and is distributed around zero under the null hypothesis of a unit root in the series. The Phillips-Perron Z_{PP}-statistic is a stationarity test that is robust to heteroscedasticity. The Engle $T \times R^2$-statistic from VAR(5) estimation residuals has an asymptotic $\chi^2(5)$ distribution under H_0 of no second-order dependence. The critical values for the Ljung-Box/Granger-Haugh/Engle $T \times R^2$ test are 78.9/51.9/9.3, 83.7/55.8/11.2, and 93.3/63.3/15.2 computed from the CDF of the $\chi^2(64)/\chi^2(40)/\chi^2(5)$ distribution, respectively. The critical values for both the Dickey-Fuller and Phillips-Perron test are −5.7, −8.1, and −13.8.

[a–c]Significant at the 10%, 5%, and 1% levels, respectively.

[d]Approximate critical values at maximum gap frequency for D.

normality are well into the critical regions for the χ^2 distribution with 2 degrees of freedom. Kolmogorov-Smirnov D-statistics, constructed from prewhitened ARMA residuals, lead to a rejection of unconditional normality as well. These findings of leptokurtosis and non-normality results are typical for empirical distributions of many speculative price series and are necessary to the MDH hypothesis (Westerfield, 1977). Ljung-Box Q-statistics for the test of white noise, performed on the same prewhitened residuals for 64 lags, reject the null hypothesis of uncorrelated increments for all contracts. This may be consistent with second-order dependence, which often leads to a spurious rejection of the random walk after controlling for first-order dependence. Q-statistics for the tests of Granger causality, constructed from cross-correlations with the volume series at ± 20 lags, reveal that in four of six cases the null hypothesis of no Granger feedback between these series (i.e., at least one series improves forecasts of the other) can be rejected, which supports the joint modeling of price change and volume. It should be noted that the series which exhibit the least Granger causality are the contracts which have generated the smallest volume (see Table II). As mentioned earlier, the Treasury bond and Eurodollar contracts have been delisted; investors prefer to trade these contracts in Chicago, where there is much more liquidity. Both Treasury bond and Eurodollar display relatively small daily logarithmic volume for the period. Low volume and lack of interest from market participants might explain the lack of Granger causality. The process is found to satisfy the necessary condition for stationarity in all cases, by an examination of Dickey-Fuller and heteroscedasticity consistent Phillips-Perron Z-statistics, which strongly reject a unit root. Finally, the Engle $T \times R^2$ statistics (Engle, 1982) for the presence of ARCH effects, computed from VAR(5) regressions of squared prewhitened residuals, strongly reject the hypothesis of no second-order dependence in all cases. The results for the natural logarithm of trading volume, reported in Table II, are similar. There is significant excess skewness and kurtosis in all cases, although the former is more consistently positive, and the latter is much less compared to the price change series. J-statistics strongly reject normality in all cases, although D-statistics reject at better than the 10% significance level in only half the cases. In every contract, the Q, Z, and $T \times R^2$ tests strongly reject white noise, a unit root, as well as no ARCH effects, respectively.

The results of the full-maximum likelihood estimation of the bivariate GARCH model are presented in Table III. Most coefficients are individually significant by asymptotic t-statistics, and jointly significant by the Lagrange multiplier statistic for the full model. Important findings

TABLE III

Maximum Likelihood Estimates of Bivariate GARCH-in-Mean Model for Financial Futures Daily Closing Log Price Change and Log Volume for Near Contract

$$\Delta f_t = \alpha_0 + \alpha_1\sqrt{h_t^f} + u_t^f \; v_t = \beta_0 + \beta_1 v_{t-1} + \beta_2 u_{t-1} + \beta_3 t + \beta_4\sqrt{h_t^v} + u_t^v$$

$$(u_t^f, u_t^v)^T \sim N((0,0)^T, H_t), \; (h_t^f, |h_t^{fv}|, h_t^v)^T = \mathrm{vech}(H_t) \; h_t^f = \exp\,(\gamma_0 + \gamma_1 \ln(h_{t-1}^f) + \gamma_2 u_{t-1}^f + \gamma_3 |u_{t-1}^f| + \gamma_4 v_{t-1})$$

$$h_t^v = \exp(\delta_0 + \delta_1 \ln(h_{t-1}^v) + \delta_2 u_{t-1}^v + \delta_3 |u_{t-1}^v| + \delta_4 \Delta f_{t-1}) \; |h_t^{fv}| = |\varepsilon_0| \exp\left(\varepsilon_1 \ln(|h_{t-1}^{fv}|) + \varepsilon_2 u_{t-1}^{fv} + \varepsilon_3 |u_{t-1}^{fv}| + \varepsilon_4 I_{\Delta f_{t-1} v_{t-1}} \sqrt{|\Delta f_{t-1} v_{t-1}|}\right)$$

$$L(\theta|Y,u) = -\tfrac{1}{2} \overset{T}{\underset{t=0}{}} (\ln(2\pi) + \ln|H_t| + u_t^T H_t^{-1} u_t)$$

where Δf_t is the log-relative daily closing price, v_t is the natural logarithm of daily trading volume, u_t^f and u_t^v are the respective random disturbances to the conditional means of price and volume, h_t^f and h_t^v are the respective conditional variances of price and volume, h_t^{fv} is the conditional covariance between price and volume, H_t is the conditional variance-covariance matrix, vech(.) is the vectorization operator that stacks the unique elements of a symmetric matrix, $I_X = \mathrm{sign}(X) \times 1$ is the indicator function, $u = (u_0, \ldots, u_T)^T$ is the random matrix of disturbances, $Y = (Y_0, \ldots, Y_T)^T$ is the time series of observations, θ is a vector of parameters, and $L(. \, .)$ is the log-likelihood of the latter conditional on (u,Y). The estimation is performed using the BHHH algorithm. Estimates are reported in the rows with asymptotic t-statistics beneath. The diagnostic statistics, D, Q_{LB}, Z_{PP}, and $T \times R^2$, test the null hypotheses of normality, white noise, unit root, and no ARCH effects, respectively, for the standardized residuals of the estimation.

Underlying Coefficient	*Sterling*	*Eurodollar*	*U.S. Treasury Bond*	*Bund*	*ECU*	*Euromark*
	0.0185	0.0835	0.6368	−.2120	0.1343	−1.8270
α_0	5.7098[c]	58.755[c]	19.620[c]	−51.311[c]	0.0955	−358.8[c]
	−1.9140	−3.6230	−21.694	−2.8980	1.8605	−2.1190
α_1	−27.761[c]	−47.859[c]	−18.504[c]	−52.858[c]	1.3218	−31.927[c]
	0.0034	−407.09	−1.3951	−103.61	1.8605	−3222.4
β_0	38.472[c]	−1.6073	−40.841[c]	−230.13[c]	7.7368[c]	−21.267[c]
	0.9810	0.1630	0.0209	0.1170	0.0553	0.7580
β_1	6.8223[a]	73.984[c]	5.0823[c]	186.09[c]	8.2230[c]	1630.1[c]
	−0.9960	−0.8609	−0.9072	−12892.3	0.9858	−0.8636
β_2	−3.9339[c]	1.6435[a]	−3.7869[a]	−190.92[a]	2.9959[b]	21.256[c]
	4.14×10^{-5}	9.46×10^{-5}	1.10×10^{-4}	1.55×10^{-5}	7.59×10^{-4}	2.07×10^{-4}
β_3	1.65.730[c]	244.13[c]	66.43[c]	227.73[c]	239.63[c]	88.730[c]
	8.93×10^{-8}	9.6800	0.3024	16.541	1.0795	0.3024
β_4	26.759[c]	68.034[c]	4.8918[c]	313.19[c]	3.7509[b]	4.8918[c]

TABLE III (Continued)

Maximum Likelihood Estimates of Bivariate GARCH-in-Mean Model for Financial Futures Daily Closing Log Price Change and Log Volume for Near Contract

	−0.4070	0.3680	−7.0906	0.0019	−0.0001	−0.0015
γ_0	−16.082[c]	6.7356[c]	−71.604[c]	97.984[c]	−13.348[c]	−73.226[a]
	0.9700	0.9364	0.8316	0.9398	0.8566	0.8212
γ_1	324.23[c]	4.7599[c]	10.213[c]	170.02[c]	54.103[c]	274.08[c]
	−0.9550	−0.9542	−0.6914	−0.0003	−0.9106	−0.8042
γ_2	−25.570[c]	5.9543[c]	−9.7636[c]	63.316[c]	2.4779[b]	73.034[c]
	0.7640	0.6012	0.5494	0.0007	0.8891	0.6327
γ_3	20.594[c]	6.4743[c]	5.8936[c]	67.836[c]	2.8749[c]	70.614[c]
	0.2990	0.0165	0.0143	0.0001	0.0001	0.0015
γ_4	18.752[c]	6.3311[c]	36.562[c]	219.01[c]	17.972[c]	69.393[c]
	−2.6420	0.0104	−0.6741	0.0079	0.0044	0.0256
δ_0	−122.01[c]	320.26[c]	24.331[c]	484.83[c]	27.598[c]	69.393[c]
	0.1410	0.0150	0.0482	0.1270	0.0284	0.0029
δ_1	20.956[c]	44.567[c]	11.746[c]	272.57[c]	4.3429[c]	58.352[c]
	38.017	0.0028	0.0137	0.0043	0.4090	0.3400
δ_2	300.93[c]	1637.3[c]	67.786[c]	1599.6[c]	170.81[c]	143.25[c]
	38.093	0.0027	9.97×10^{-4}	1.94×10^{-5}	0.4089	0.3411
δ_3	300.86[c]	−11.699[c]	−65.618[c]	219.01[c]	9.8872[c]	41.326[c]
	0.0137	0.0092	3.6424	0.0853	0.1755	0.2400
δ_4	43.822[c]	82.822[c]	65.222[c]	94.220[c]	109.022[c]	51.122[c]
	0.0027	0.0569	3.6424	0.0026	0.0004	0.0029
ε_0	24.804[c]	13.270[c]	65.618[c]	600.02[c]	6.4677[c]	125.14[c]
	−0.0084	−0.3314	−0.0722	−0.0014	−0.0001	−0.4290
ε_1	−4.3081[c]	12.706[c]	−6.8627[c]	491.97[c]	−0.3260[c]	127.69[c]
	2.3790	0.0010	−7.4733	0.1010	0.5107	0.0232
ε_2	5.6560[c]	8.4400[c]	−6.4403[c]	235.30[c]	30.937[c]	163.30[c]
	−2.4680	−0.0019	7.5183	−0.1055	−0.5721	−0.0318
ε_3	−5.8638[c]	−7.9640[c]	6.8063[c]	−222.90[c]	−27.937[c]	−205.80[c]
	−2.2790	0.0376	−7.4733	−0.0013	−0.0016	−0.0134
ε_4	−40.819[c]	12.984[c]	−8.3910[c]	−254.92[c]	−10.739[c]	−108.04[c]
logl	−33109.7	−485868.8	−57290.3	−221540.4	−17012.82	−12108.93
$D\ \Delta f_t$	0.0128	0.0297	0.0296[a]	0.0292	0.0297	0.0336
v_t	0.0195	0.0173	0.0186	0.0204	0.0249	0.0240
$Q_{LB}\ \Delta f_t$	46.68	78.15	73.30	72.83	45.79	48.95
v_t	64.17	49.61	62.63	55.76	92.99[b]	46.79
$Z_{PP}\ \Delta f_t$	−2943.6[c]	−2852.4[c]	−2155.4[c]	−1473.46[c]	−1132.1[c]	−1294.7[c]
v_t	−2900.2[c]	−2883.8[c]	−2096.3[c]	−1456.5[c]	−1174.6[c]	−1265.9[c]
$T \times R^2$						
Δf_t	1.023	11.865[b]	2.641	0.684	3.755	7.243
v_t	3.112	2.378	1.954	2.369	1.478	5.912

[a–c]Significant at the 10%, 5%, and 1% levels, respectively.

include the significant GARCH found in all series, with statistically significant estimates of magnitudes suggestive of stability and stationarity in the second moments. Particularly important is the positive and statistically significant scale coefficient, ε_0, on the conditional covariance in all six cases. The GARCH coefficients (γ_i, δ_i, and ε_i for $i = 1, \ldots, 4$) are statistically significant as well, implying that the covariance exhibits time variation. This result is at odds with the typically insignificant correlations that have been found in futures market data between price changes and level of trading activity (Karpoff, 1987). This can be interpreted as more favorable to the MDH than certain transactions cost models (Copeland, 1976) that attribute the lack of correlation in derivatives markets to the absence of frictions such as short sale restrictions. The results of this study can be explained also by noisy rational expectations equilibrium models, where volume is informative about the price signal distribution (Blume et al., 1994; Easley et al., 1994), as well as the price pressure arguments of technical analysts (Murphy, 1986; Kaufman, 1987) that are consistent with these models. Note that the coefficients, ε_1, on the lagged conditional covariance are negative, implying mean reversion toward a level of unconditional mean covariance. Looking at the terms, ε_2 and ε_3, for the innovation cross-product, it is evident that the conditional covariance is higher for innovations of opposite sign. This means that one variable becomes a better forecasting tool for the other when it is at relatively more abnormal levels than the other (e.g., it is easier to forecast the price using volume in a period of moderate prices but high trading activity).

Considering the effect of volume on the conditional variance of price change, it is found that trading volume positively influences the conditional heteroscedasticty of return, in that the coefficient estimates of γ_4 are positive and statistically significant in all six cases. These results are similar to the findings of Najand and Yung (1991), who found lagged volume to have explanatory power for price volatility in Treasury bond futures, but differ from the results of Lamoureux and Lastrapes (1990), who found that ARCH effects disappear when volume is added as a regressor for equity indices. This can be interpreted, consistent with the MDH, as information flow about interest rate movements being reflected in a volume proxy that is positively related to the futures price conditional variance. A related result concerns volume conditional variance, which is found to be increasing in price change, by the statistical significance of δ_4 in all cases. This can be interpreted as consistent with Karpoff's (1987) asymmetry hypothesis for volume, in which its volatility and expected level should be higher on price up-ticks than on price down-ticks.

Examining the dynamics of the conditional second moments of return, all contracts are found to have positive and significant coefficients on the lagged conditional heteroscedasticity of return. In terms of persistence in volatility, this implies that a shock to the conditional variance has a half-life of approximately 22.75 trading days.[12] The only cause for concern is that the magnitudes are all on the order of 0.9, which could say that there is too much persistence in this component of the variance for stability. However, note that in all cases one would reject the hypothesis that the parameters $\gamma_1 = 1$. Second, the Regression Analysis of Time Series (RATS) software has built-in checks on the magnitude of the GARCH parameters for stability and stationarity of the coefficient estimates (Doan, 1992).[13] Finally, the unit root tests for the squared process suggest that a unit root is not present here.[14]

The coefficient estimate of α_1, which measures the sensitivity of price change to time variation in the risk premium, is negative and significant in five of six contracts (the exception being ECU). This is fully consistent with the findings in studies of various markets such as equities (French, Schwert, and Stambaugh, 1987), interest rates (Engle et al., 1987), currencies (Hsieh, 1989), as well as for commodity futures (Dusak, 1973). These results can be interpreted as a relationship between unanticipated changes in interest rates (a measure of systematic risk) and expected futures prices changes as specified in eq. (1).

The next result concerns asymmetry in conditional price change variance to innovations. In five of six contracts, $\gamma_2 < 0$, $\gamma_3 > 0$, and $|\gamma_3| < |\gamma_2|$, at significant probability values. This means that volatility is monotonically declining in price innovations, with negative futures price surprises increasing volatility by a greater amount in absolute terms than positive innovations of equal magnitude. Only in the case of Bund is $|\gamma_3| > |\gamma_2|$, which means that volatility increases in positive innovations. Except for the latter case, these results are at variance with Nelson's (1991) finding, for equity index spot returns, of an asymmetric V-pattern for the plot of volatility against price surprise. Unlike the positive covariance

[12]This is the number, h, such that $\rho^h = 1/2$, where ρ is the first-order autocorrelation coefficient on h_t^f. This is only meant to be suggestive. The precise dynamics in a bivariate case are beyond the scope of this paper.

[13]However, these checks are sufficient neither to insure that the unconditional variance will be well defined, nor that the conditions for the existence of higher-order moments are satisfied. The former is one reason why exponential GARCH is employed (Nelson, 1991), while the latter is not a concern in maximum likelihood estimation (Hsieh, 1989). The exact conditions for stability and stationarity in this context are beyond the scope of this paper.

[14]Cai (1994) argues that the appearance of a unit root in the variance may be a result of not accounting for regime shifts in the econometric model, which is a possibility in these data given the length of some of the series.

between price changes and volume, these results differ from this V-shape stylized fact of equity markets. This raises several questions, the principal being how this negative relationship between innovations and conditional variance for $u_t^f > 0$ can be explained. Since the leverage effect explanation of Black (1976) and Christie (1982) is clearly inapplicable to interest rate futures, the findings of this study may lend support for their rationale. On the other hand, this finding calls for an explanation in the context of interest rate futures.

Estimation results for the dynamics of mean log-volume and log-conditional variance of volume reveal persistence at both levels. The autocorrelation coefficient in the mean equation of volume, β_1, is significantly different from zero and positive for all contracts. The estimates are very precise, and in all cases, the unit root hypothesis is rejected. Nevertheless, there is still much persistence in volume, in that for $\beta_1 = 0.5$, abnormal surges in volume persist for about 35 trading days, which is only about 1/40 times the length of the average sample. A potentially more serious problem that could make one question these point estimates is the magnitude of the moving average coefficients. For five of six cases they are significantly negative and close to unity (the p-value is between 5% and 10% for Eurodollar), even though the null hypothesis of a unit root can be rejected in all cases. However, inverse autocorrelations of volume[15] decay rapidly, which is favorable to the invertibility of the process. The time trend coefficients are significantly positive, as expected, given the growth of these markets during the period under study. Finally, for the mean equation, the GARCH-in-mean coefficients are significantly positive in all cases. This finding, along with the significant GARCH effects, is consistent with the simulation result of Karpoff (1986). However, the level of persistence in the volatility of volume, as measured by the coefficient δ_1, is much lower than that of the volatility of returns. The coefficients are significantly positive in all cases, yet on the order of 1/100th to 1/10th, implying shock half-lives of about one third of a trading day.

The results show a peculiar type of innovation asymmetry in the conditional variance of volume, in that it is conditionally rising in positive innovations, but affected by those of negative sign. This can be seen by examining the coefficients, δ_2 and δ_3, on u_t^v and $|u_t^v|$, respectively, and noting that in four of six cases they are statistically indistinguishable. An explanation is that it is only *excess* volume that is the proper proxy for the directing variable for the subordinated joint distribution of volume and

[15]Not reported here but available upon request.

price. This is clearly related to the prediction of positive skewness in the empirical distribution of volume predicted by the MDH (Clark, 1973), in that if positive errors are associated with augmented volatility, then the persistence in volatility gives rise to such a distribution.

Residual diagnostics for the standardized GARCH errors (using a Cholesky decomposition with the conditional moments) are presented in the final rows of Table III. They consist of Kolmogorov-Smirnov D-statistics for normality, Ljung-Box Q-statistics (64 lags) for white noise, Phillips-Perron (heteroscedasticity consistent) Z-statistics for a unit root, and Engle $T \times R^2$ tests (at 5 lags) for ARCH effects. The null hypothesis of normality is not rejected at greater than 10% p-values in five of six cases (for U.S. Treasury bond the p-value is between 10% and 5%). The standardized residuals are found to be uncorrelated in all cases at p-values greater than 10%, except for a rejection at this level for ECU volume residuals. In all cases the hypothesis that the errors have a unit root is rejected. Finally, with the exception of Eurodollar price change residuals, no ARCH effects are found in these whitened series. The conclusion is that this bivariate GARCH(1,1) model adequately captures the dynamics of these data.

CONCLUSIONS

The findings of this study can be summarized as follows. First, there is strong evidence of second-order dependence in the joint futures return and trading volume process for various international financial futures markets. The estimation also reveals significantly positive, contemporaneous time varying covariance between these series. The latter result is new to the price-volume literature, insofar as futures data are concerned (Karpoff, 1987). Second, the study has shown evidence that the level of trading volume positively influences the conditional variance of the futures price change. This is in line with Najand and Yung's (1991) finding, with the difference being that they examined Treasury bond futures trading domestically on CBOT, while this study looked at a cross-section of financial futures trading on LIFFE. In addition, it is shown that the conditional return on holding several international futures contracts exhibits an inverse relationship to a measure of systematic risk, as proxied by its conditional variance. This result is similar to the finding of Kroner and Lastrapes (1993) for exchange rate data. Finally, the paper records a significant asymmetric response of return volatility to innovations in return, although this pattern is monotonically decreasing, in contrast to the V-shape documented for equity return data (Nelson, 1991).

The findings of this study are relevant to the study of interest rate futures, in particular, and speculative markets, in general. There are market efficiency implications arising from these results suggesting that these futures prices exhibit second-order dependence, with the variance a function of trading volume, in support of the MDH. If volume is taken to be a proxy for, or a function of, a latent information variable, then the MDH is at odds with market efficiency in the sense that there exists a *positive* expected price response to information arrival for these contracts. The fact that trading frictions are low in futures markets relative to spot markets makes this all the more surprising. The second area of importance is to those who are concerned with how agents in speculative markets construct optimal forecasts, as well as those who use variance estimates for academic exercises. This research presents evidence that optimal volatility forecasts are necessary to forecast futures prices and that these are both time varying and connected to trading volume, which may lead to a reconsideration of past methodologies utilizing volatility measures. Interestingly, these results have the implication that popular volume-based technical analysis rules used in futures markets (Murphy, 1986; Kaufman, 1987) might not be ill defined. In particular, the notion that volume is a measure of price pressure and can therefore forecast changes in the direction of price trends, is supported by our results (Murphy, 1986).[16] The third contribution of this research lies in the domain of price discovery. The way in which these prices evolve and agents construct forecasts has a direct bearing upon the price formation process in spot markets. This allows researchers to pose questions of relevance to policy, such as the influence of futures market activity upon price volatility in the spot markets. Finally, it should be noted that, in general, the issue of time varying volatility is of importance to option pricing. The implication of these findings that futures price changes and volume are not only jointly distributed, but that volume influences price volatility, can guide theorists and practitioners alike in rethinking the pricing relationships for financial futures.

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[16]For interest rate futures, *lagged* movements in volume are associated with subsequent movements in rates, in that the former is directly related to the conditional volatility of futures price changes, and the latter is inversely related to the price changes through the GARCH-in-mean and therefore directly related to (and in this sense "support") interest rates.

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