The Journal of Risk

Journal of Risk 21(5), 1–24
DOI: 10.21314/JOR.2019.410

Research Paper

A generic stress testing framework with related economic shocks and possible regulatory intervention

Dror Parnes[1] and Michael Jacobs Jr.[2]

[1]Economics and Finance Department, College of Business, BA204, Texas A&M University-Commerce, Commerce, TX 75429-3011, USA; email: dror.parnes@tamuc.edu

[2] PNC Financial Services–APM Model Development, 340 Madison Avenue, New York, NY 10022, USA; email: michael.jacobsjr@pnc.com

(Received August 31, 2017; revised June 11, 2018; accepted June 27, 2018)

ABSTRACT

In this study, we develop and demonstrate a universal framework for supervisory stress tests of financial institutions that considers the probable dependencies among macroeconomic shocks and possible regulatory intervention. The proposed differential equations model can assess the combined influence of related shocks in various markets and economic attributes on banks' excess capital beyond minimum regulatory ratios. The suggested model allows policy makers to implement sensitivity analyses, which reveal how an examined bank's excess capital would react to diverse economic shocks with a wide range of varying intensities. Our model can further assess the likely impact of regulatory intervention at different magnitudes and at various points in time. It can therefore help regulators to select the optimal intervention in different economic settings.

Keywords: stress tests; Comprehensive Capital Analysis and Review (CCAR); banking; related shocks; regulatory intervention.

Corresponding author: D. Parnes
Print ISSN 1465-1211 | Online ISSN 1755-2842

1 INTRODUCTION

In this study, we develop a generic framework for supervisory stress tests of bank holding companies (BHCs) that takes into account the genuine dependencies (typically observed and processed from past records, but which can also be modified based on future trajectories or personal opinions) among macroeconomic shocks and possible regulatory intervention. The proposed differential equations model can measure the combined effect of related shocks in various markets and economic attributes on BHCs' excess capital beyond minimum regulatory ratios. The suggested model allows policy makers to implement sensitivity analyses, which reveal how an examined BHC's excess capital would react to diverse economic shocks with a wide range of varying intensities (and not just to three basic scenarios). Moreover, our model can further assess the probable impact of regulatory intervention at different magnitudes and at various points in time. It can therefore help regulators to select the optimal intervention under different economic circumstances.

Stress testing became mandatory in 2004 following the introduction of the Basel II Accord, a support tool for both validating capital adequacy among individual BHCs and assessing the systemic risk in corresponding banking industries. Various institutions are now required to conduct periodic stress tests. The World Bank and the International Monetary Fund (IMF), for example, deploy annual "macro stress tests" to different banking systems worldwide.[1] In the United States, the Federal Reserve and the Office of the Comptroller of the Currency (OCC) execute annual "supervisory stress tests" to BHCs with total assets of US$50 billion or more, as well as to some nonbank financial companies. The Dodd–Frank Wall Street Reform and Consumer Protection Act from 2010 also requires overseen BHCs and banks with total assets of US$10 billion or more to implement their own "company-run stress tests" twice a year, and to report their results to the regulatory bodies. The importance of these stress tests has been accentuated by the financial crisis of 2007–9.

Supervisory stress tests essentially simulate how well individual banks will endure hypothetical future macroeconomic scenarios. These adverse scripts are not in any way projections; they merely incorporate extreme yet plausible economic shocks. For instance, as dictated by the Dodd–Frank Act, the Federal Reserve alternates trajectories for twenty-eight macroeconomic variables along three general settings (baseline, adverse and severely adverse).[2] The Federal Reserve then simulates how these economic shocks would affect individual banks' balance sheets, risk-weighted assets,

[1] An introduction, overview and comparative analysis of different methodologies for macro stress testing can be found in, among others, Goodhart *et al* (2004), Sorge (2004), Goodhart (2006), Goodhart *et al* (2006), Sorge and Virolainen (2006), Čihák (2007), Kida (2008) and Wong *et al* (2008).

[2] These twenty-eight variables include sixteen general variables in three categories, as follows. The first category consists of US economic activity measures (real gross domestic product (GDP)

income statements, capital levels and four regulatory capital ratios (common equity tier 1 ratio (CET1), tier 1 risk-based capital ratio, total risk-based capital ratio and tier 1 leverage ratio) over a nine-quarter standard "planning horizon".

Within these supervisory bank stress tests, each of the three simulated economic scenarios (baseline, adverse and severely adverse) contains specific hypothetical paths for the twenty-eight macroeconomic variables along the planning horizon, where the degrees of economic shocks are arbitrarily selected.[3] For example, in Board of Governors of the Federal Reserve System (2016a), the severely adverse scenario depicts an increase of 5% by the middle of 2017 (to a level of 10%) in the unemployment rate, where the consumer price inflation rises from about 0.25% in the first quarter of 2016 to an annual rate of 1.25% by the end of the recession. In this severely adverse scenario, equity prices are assumed to fall by approximately 50% through the end of 2016, housing prices to drop 25% by the third quarter of 2018, and commercial real estate prices to drop 30% through the second quarter of 2018. In contrast, in the same publication, the adverse scenario depicts an increase of 2.5% by the middle of 2017 (to a level of 7.5%) in the unemployment rate, where the consumer price inflation rises to an annual rate of 1.75% by the first quarter of 2019. In this adverse scenario, equity prices are assumed to fall by roughly 25% through the end of 2016, housing prices to drop 12% by the third quarter of 2018, and commercial real estate prices to fall 12% through the third quarter of 2017.

Upon completion of the supervisory stress tests, and once the inclusive results for the stress tests are accumulated for each of the BHCs, the Federal Reserve deploys its annual Comprehensive Capital Analysis and Review (CCAR) program. This is a relatively new regulatory inclusive agenda that considers both qualitative and quantitative measures.[4] It assesses, regulates and monitors BHCs in terms of their overall

growth, nominal GDP growth, real disposable income growth, nominal disposable income growth, unemployment rate and consumer price index (CPI) inflation rate). The second category consists of asset prices in the main financial markets (Dow Jones total stock market index, house price index, commercial real estate price index and market volatility index). The third category consists of various interest rates (three-month Treasury rate, five-year Treasury yield, ten-year Treasury yield, BBB corporate yield, mortgage rate and prime rate). An additional twelve variables assess the real GDP growth, the inflation rate and the US/foreign currency exchange rate in each of the following four international markets: the eurozone, Asia, Japan and the United Kingdom.

[3] According to Board of Governors of the Federal Reserve System (2016a), the approach to modeling their stress tests reflects "an independent supervisory perspective", and the designated economic trajectories are "forward-looking and may incorporate outcomes outside of historical experience".

[4] The CCAR program was first launched in 2011 and deployed primarily for sensitivity analyses. It replaced the Supervisory Capital Assessment Program (SCAP) from 2009. It was only in 2012 that CCAR started being used as one of the key inputs in decision making for the Federal Reserve and hence disclosed publicly.

capital policies and adequacy. Within this program, regulators can approve or reject future planned capital distributions, such as dividends, share repurchases or executive bonuses. Under some circumstances, they may also restrict firm-wide practices (eg, lending to specific segments of the market) or even recommend emergency capital injections to BHCs by the Federal Reserve.[5] Naturally, because of possible regulatory intervention, the results of the supervisory stress tests could be substantially different pre-CCAR and post-CCAR. The CCAR program aims to ensure that each BHC "maintains post-stress capital ratios that are above the applicable minimum regulatory capital ratios in effect during each quarter of the planning horizon".[6]

In practice, economic shocks are certainly not mutually exclusive. They are often related and may trigger each other with either partial or full intensity. For example, as Sorge (2004, p. 5) notes:

> An oil price shock is likely to have repercussions on inflation and interest rates and therefore can be a source of interest rate risk as well as credit risk, commodity price risk, etc.

Kida (2008) also refers to "the transmission of shocks through various channels" as a "feedback effect". Ramey (2016) discusses further paths on which macroeconomic shocks often propagate. We therefore lean on this common phenomenon of contagious tremors and accentuate in our proposed model the likely proliferation of macroeconomic shocks. These cross-dependencies can activate time-lagged economic shocks or simultaneous reactions.

In addition, as observed during the recent financial crisis of 2007–9, regulatory intervention is both a legitimate and an effective technique to mitigate structural vulnerabilities and overall risk exposures among BHCs. These occasionally vital interferences may appear as proactive measurements in advance (in light of the CCAR program) or as emergency rescue actions once financial distress or liquidity problems have already emerged. We therefore embed in our suggested framework hereafter the option for a regulatory intervention at different magnitudes and in various stages of jeopardy.[7]

[5] Institutions that require emergency capital injections conventionally enter into special commitments to issue convertible preferred securities to the US Treasury. These institutions will get temporary permissions (up to six months) to raise private capital in public markets to meet their regulatory required minimum capital and would be able to abandon their commitments without any penalties.

[6] Interested readers can find more information on the recent CCAR program in Board of Governors of the Federal Reserve System (2016b).

[7] Although regulators do not allow banks to incorporate possible bailouts into their models of company-run stress tests, a later subsection on possible regulatory intervention (which can be interpreted along the lines of BHCs' self-intervention as well) is applicable in the broader CCAR framework.

The contributions of the present study cover several dimensions. We present a generic framework for supervisory stress testing that regulators and policy makers can easily deploy. The proposed model is highly adaptable and, in contrast to current practices, where only three scenarios are customarily examined (baseline, adverse and severely adverse), our model can yield a large spectrum of outcomes based on vastly tunable macroeconomic shocks. The stress testing framework hereafter contemplates mutual dependencies, often observed, among economic variables. Further, the suggested model allows supervisors and policy makers to examine the likely impact of diverse regulatory intervention. It thus assists them in selecting the optimal interference, if needed, at the CCAR stage.

This study proceeds as follows. In Section 2, we provide a concise review of the economic literature on individual bank stress tests. In Section 3, we assemble our proposed framework, validate its underlying assumptions and extend it to further portray the inclusive dynamics involved as well as embed possible regulatory intervention. In Section 4, we illustrate the various modules of the model with four notional yet representative numerical examples. In Section 5, we conclude and point to future related lines of research.

2 RELATED LITERATURE

Many studies already explain the purpose, broad usage and added value of individual banks' stress tests as a complementary tool in the hands of policy makers. We therefore provide only a brief summary of some of the recent milestones in the literature that have offered different techniques for stress tests.

The economic literature presents various approaches to tackling the stress testing of individual banks and bank loan portfolios. Some scholars advise employing value-at-risk (VaR) and extreme value theory (EVT) methodologies as the principal framework for stress testing. Among them, Dimson and Marsh (1997) utilize the worst outcome of a portfolio value to compute the risk of a specific position. Jackson *et al* (1997) examine the empirical performance of different VaR models using data on the actual fixed income, foreign exchange and equity security holdings of a large bank. Kupiec (1998, 1999) offers generic methodologies that parameterize stress tests' scenarios using the conditional probability distributions typically used in VaR applications. Kupiec also demonstrates that his loss exposure measures (similar to our approach, these measures are consistent with historically observed volatility and dependency patterns) have superior accuracy to other popular measures. Longin (2000) further suggests that stress tests should extract the limiting distributions of extreme value computations (minimum and maximum return observations over a given time period). Tan and Chan (2003) elaborate on these methods and examine whether the underlying assumption of normality is adequate for stress testing under StressVaR and StressVaR-x procedures.

Several researchers recommend relying on credit ratings or market risk measurements. Bangia *et al* (2002) link credit migration matrixes to contractionary and expansionary business cycles as a conceptual framework for stress testing credit portfolios. Alexander and Sheedy (2008) introduce a novel methodology for stress testing that can incorporate both volatility clustering and heavy tails, evaluating the performance of eight market risk models. The latter authors realize, however, that stress test results should hold little sway over the levels of foreign exchange regulatory capital.

Both Berkowitz (1999) and Greenspan (2000) criticize current methodologies of stress testing in light of their lack of precision and consistency. These authors claim that existing stress testing techniques usually examine adverse scenarios but without any matching likelihoods or an overlying structure; thus, comprehensive risk evaluations for the tested banks are problematic. Consequently, some studies recommend bonding banks' stress test scenarios through more intuitive statistical models (our present approach is such a model). Berkowitz (1999), for example, proposes combining economic scenarios with their corresponding probabilities, which can be inferred from past events. Rebonato (2010) suggests a Bayesian approach as an overlying configuration for stress tests' scenarios. Aragones *et al* (2001) integrate banks' stress tests into a formal market risk model. Jacobs *et al* (2015) further utilize a Bayesian approach to stress test credit risk portfolios. Kopeliovich *et al* (2015) take the reverse approach and show how to select the most likely stress test scenario for generating a specific capital loss.

Other studies stress test retail loan portfolios in particular. Among them is Kearns (2004), who documents this application using Irish retail credit institutions and finds some evidence that the level of loan losses, judged by loan-loss provisions, rises when GDP growth declines, but even more significantly when unemployment escalates. Rösch and Scheule (2007) develop a framework to stress test the smallest building blocks of a portfolio, ie, the loans themselves, while accounting for the cross-correlations in each portfolio. Breeden *et al* (2008) further present a vintage approach for stress testing retail loan portfolios, which is based on the common origination time for the specific loans within.

Other articles combine different sources of risk as their core methodologies for stress testing. Drehmann *et al* (2010) measure credit and interest rate risks jointly and show how together they affect banks' economic values and capital adequacies. Similar to our journey hereafter, these authors also stress test a hypothetical but realistic bank to illustrate the functionality of their proposed related-shocks model. Fender *et al* (2001), Foglia (2009) and Jacobs (2013) provide surveys on various approaches to bank stress testing.

3 THE MODEL

3.1 The building blocks for the model

To formulate the settings at which regulatory bank stress tests are typically implemented, we denote $\beta > 0$ as the dollar amount of excess capital that an individual BHC currently (pre-CCAR) has above the minimum regulatory threshold (positive for all active BHCs). We assign $\alpha_j \geqslant 0$ to represent the dollar amount of direct financial damage to the BHC's capital from economic shock j (along the $n = 28$ macroeconomic variables) as captured by the Federal Reserve proprietary models.[8] In this case, the ratio

$$\gamma_j = \frac{\alpha_j}{\beta} \in [0,1], \quad j = 1, 2, \dots, n, \tag{3.1}$$

represents the total percentage of monetary damage (or loss) to the BHC caused by economic shock j and scaled by the excess capital this BHC currently holds above the minimum regulatory threshold.[9] These ratios are simulated, and, thus, we treat them in our later numerical illustrations as observable. In reality, any adverse economic shock would cause some harm to a BHC's capital, except perhaps for monoline credit card companies with little or no exposure at all to most economic sectors. However, this isolated damage, in most situations, would not exceed the entire buffer zone that BHCs normally maintain above the minimum regulatory threshold; thus, we deem γ_j to be nonnegative ($\gamma_j \geqslant 0$) and at the same time capped at 100% ($\gamma_j \leqslant 1$). These assumptions, however, are not mandatory for our mathematical formulation hereafter.

Because, in reality, most economic variables are related, at least to some degree, and shocks along the twenty-eight macroeconomic variables should never occur in complete isolation, we let λ_{kj} be the degree of dependence between economic shocks j and k. Depending on the application and the desired causality across these shocks, this type of reliance could be time-lagged (and, in this case, $\lambda_{k_t j_{t-1}} \neq \lambda_{j_t k_{t-1}}$)

[8] Board of Governors of the Federal Reserve System (2016a) describes its stress tests as assessing how the simulated economic shocks are projected to affect BHCs' "revenue expenses, and various types of losses and provisions that flow into pre-tax net income based on data provided by the BHCs and on models developed or selected by Federal Reserve Staff and evaluated by an independent team of Federal Reserve model reviewers". These models "take into account the US Generally Accepted Accounting Principles (GAAP) and regulatory capital rules", and further "consider the characteristics of the BHCs' loans and securities portfolios; trading, private equity, and counterparty exposures from derivatives and SFTs; business activities; and other relevant factors".
[9] For example, Board of Governors of the Federal Reserve System (2016a) reports that in the adverse scenario, the aggregate CET1 ratio would fall from 12.3% to 10.5%, while in the severely adverse scenario this aggregate CET1 ratio would fall from 12.3% at the end of 2015 to a post-stress level of 8.4% in the first quarter of 2018.

or timely synchronized (hence $\lambda_{k_t j_t} = \lambda_{j_t k_t}$). Nonetheless, it is highly improbable that one economic shock deters another economic shock; thus, we shall assume the degrees of dependence between pairs of economic shocks are nonnegative, ie, we exclude the unlikely scenario that $\lambda_{kj} < 0$ and restrict $\lambda_{kj} \in [0, 1]$ for all $j, k = 1, 2, \dots, n$. These presumably nonnegative dependency measures are reasonably attainable from historical records. Although this is not the established practice, they can also be calibrated from standard actuarial models. Alternatively, they can be modified based on subjective opinions or future prospects.[10]

Following these lines, we further appoint $\gamma_{kj} \in [0, 1]$ to represent the marginal percentage of monetary damage caused by the impact of economic shock j on economic shock k (in light of their interdependency) and scaled by the excess capital the BHC currently holds above the minimum regulatory threshold. These parameters are not observable, but they are computable, as discussed hereafter.

Finally, we denote $\delta_k \in [0, 1]$ as the in-isolation percentage monetary damage (or financial loss) to the bank from economic shock k once scaled by the excess capital the BHC currently holds above the minimum regulatory threshold. These parameters are the input for the subsequent model, ie, they are arbitrarily selected and injected into our proposed scheme at various levels to create a sensitivity analysis. This analysis would reveal how much a BHC's excess capital (the buffer zone beyond the regulatory threshold) is damaged by different levels of economic shocks and what could be the likely effect of various government interferences.

We can now develop the two basic relations underlying our suggested framework. The first proportionality assumption rationally presumes that the marginal percentage of the monetary loss caused by the impact of economic shock j on economic shock k, once scaled by the excess capital that the BHC currently holds above the minimum regulatory threshold, has to be eternally equal to the product of the total percentage of damage to the BHC caused by economic shock j, once scaled by the excess capital this BHC currently holds above the minimum regulatory level, and the degree of dependence between economic shocks j and k. More formally,

$$\gamma_{kj} = \lambda_{kj}\gamma_j = \lambda_{kj}\frac{\alpha_j}{\beta} \quad \text{for all } j, k = 1, 2, \dots, n. \tag{3.2}$$

The second balancing equation logically considers that the total percentage of monetary damage to the BHC caused by economic shock k, once scaled by the excess capital this BHC currently holds above the minimum regulatory threshold, is a linear combination of all the marginal percentages of the monetary losses caused

[10] Among others, Ramey (2016) reviews narrative methods (which can identify quantities associated with a particular change in economic variables) and suggests various techniques to compute the impulse responses underlying these dependency measures.

by the impact of any other economic shocks on the economic shock k and the in-isolation percentage damage to the bank from economic shock k, both scaled by the excess capital the BHC currently holds above the minimum regulatory threshold. More formally,

$$\gamma_k = \sum_j \gamma_{kj} + \delta_k \quad \text{for all } j, k = 1, 2, \dots, n. \tag{3.3}$$

At this point, we can combine these two underlying equations and form the following single-shock relationship:

$$\gamma_k = \sum_j \lambda_{kj} \gamma_j + \delta_k \quad \text{for all } j, k = 1, 2, \dots, n. \tag{3.4}$$

In (3.4), we essentially assume that $\lambda_{kk} = 0$ for all k. In doing so, we exclude all the effects of economic shocks on themselves; we therefore add the term δ_k. This single-shock relationship is both intuitive and mathematically complete. Hence, it can be further expanded to a more general multiple-shock matrix notation that conveys the entire scope of a supervisory stress test as

$$\gamma = \mathcal{S}\gamma + \delta, \tag{3.5}$$

which can be further developed into

$$\gamma = (I - \mathcal{S})^{-1}\delta, \tag{3.6}$$

where $\gamma = [\gamma_1, \gamma_2, \dots, \gamma_n]^{\mathrm{T}}$ is a transposed row vector (ie, a column vector) that captures all the total percentages of monetary losses to the BHC caused by the different economic shocks and scaled by the excess capital this BHC currently has above the minimum regulatory threshold; $\delta = [\delta_1, \delta_2, \dots, \delta_n]^{\mathrm{T}}$ is a column vector that captures all the in-isolation percentage damages; $\mathcal{S} = [\lambda_{kj}]$ is an $n \times n$ square matrix that contains all the nonnegative dependency measures across the diverse economic shocks; and I is the $n \times n$ identity matrix.[11]

In this setting, the total (accumulated) capital loss to the BHC from the combined effect of the diverse macroeconomic shocks is simply

$$\Delta = \sum_{i=1}^{n} \gamma_i = 1 \times \gamma, \tag{3.7}$$

[11] Constructing and modifying a dependency matrix $\mathcal{S}$ with dimensions 28×28 is not a trivial process. For this task, we recommend following the guidelines and algorithms provided by Sharman and Yassine (2004), which can substantially reduce the complexity of the process by isolating subsets of the matrix elements (also called clusters, modules or chunks in the mathematical literature) that are minimally interacting.

where 1 denotes a $1 \times n$ row vector of ones. A BHC fails whenever $\Delta \geqslant 1$, ie, when the BHC's excess capital beyond the minimum regulatory threshold is completely eliminated in the supervisory stress test. In this case, the minimum capital adequacy ratio is breached and we can assume that the regulator classifies the financial institution as a failed bank.

In (3.6), we essentially assume that the determinant of $(I - \mathcal{S}) \neq 0$; therefore, this square matrix is nonsingular and can be inverted. In practice, this assumption is highly probable, since $\det(I - \mathcal{S}) = 0$ for a random square matrix $\mathcal{S}$ of dimensions 28×28 only in exceptionally rare circumstances. Equation (3.6) is also a Leontief-type input–output model, which allows us to further investigate its properties.[12]

3.2 Evaluating the assumptions

Before we examine several aspects of our framework, we need to verify that our underlying assumptions are indeed reasonable and can hold under extreme circumstances, such as when severely adverse economic shocks are simulated, ie, when δ_k are large enough, or when $\delta_k \gg 0$. Let us consider a supervisory stress test that contains n disjoint macroeconomic shocks. The total damage to the BHC's excess capital (ie, the buffer zone) would then be

$$\gamma_i = f_i(\gamma_1, \gamma_2, \dots, \gamma_n) + \delta_i, \tag{3.8}$$

where the function $f(\cdot)$ is strictly determined by the complete structure of dependencies among the different economic shocks, with the same general constraints as before: $i = 1, 2, \dots, n$ and $\gamma_i \in [0, 1]$. Endless mathematical expressions can shape function $f(\cdot)$, but we can assume this general function is continuous and sufficiently differentiable (ie, high-order derivatives exist). In addition, we can presume that when no economic shocks are simulated, the BHC suffers no monetary damage at all and its excess capital above the minimum regulatory threshold remains at the present level; therefore, function $f(\cdot)$ yields zero output, or

$$f_i(0, 0, \dots, 0) = 0 \quad \text{for all } i = 1, 2, \dots, n. \tag{3.9}$$

[12] In Leontief (1951), the author – the winner of the 1973 Nobel Memorial Prize in Economic Sciences – presents his revolutionary idea to describe the interdependencies among different sectors of the national economy through an input–output differential equations model. The Leontief system is often extended to a model of general equilibrium. Because the Leontief model is linear, it evolves toward intuitive, flexible and rapid computations. This makes the approach highly popular among policy makers and monitoring bodies. These days, the Bureau of Economic Analysis (BEA) uses Leontief-based tables (see, for example, Horowitz and Planting 2006). Both the World Bank and the IMF also utilize the Leontief framework to explore the interdependencies among worldwide economies.

It would also be rational to assume that function $f(\cdot)$ is a nondecreasing function, since, in practice, stronger economic shocks should yield to the BHC higher financial losses, which are all positively interrelated. No further assumptions concerning the convexity or the concavity of function $f(\cdot)$ are required. In this case, since the first-order derivative exists, then

$$\frac{\partial f_i(\gamma_1, \gamma_2, \dots, \gamma_n)}{\partial \gamma_j} \geqslant 0 \quad \text{for all } i, j = 1, 2, \dots, n. \tag{3.10}$$

We can now take the Taylor series expansion of f_i and obtain

$$\gamma_i = f_i(0, 0, \dots, 0) + \sum_{j=1}^{n} \frac{\partial f_i(0, 0, \dots, 0)}{\partial \gamma_j}\gamma_j + \sum_{j=1}^{n}\sum_{k=1}^{n} \frac{\partial^2 f_i(0, 0, \dots, 0)}{\partial \gamma_j \partial \gamma_k}\gamma_j\gamma_k + \cdots + \delta_i, \tag{3.11}$$

but since $f_i(0, 0, \dots, 0) = 0$ in (3.9), we get

$$\gamma_i = \sum_{j=1}^{n} \frac{\partial f_i(0, 0, \dots, 0)}{\partial \gamma_j}\gamma_j + \sum_{j=1}^{n}\sum_{k=1}^{n} \frac{\partial^2 f_i(0, 0, \dots, 0)}{\partial \gamma_j \partial \gamma_k}\gamma_j\gamma_k + \cdots + \delta_i. \tag{3.12}$$

When external economic shocks are rather large, ie, whenever $\delta_i \gg 0$, we can ignore negligible high-order terms. As a result, a first-order approximation is rightly legitimatized. In this case, (3.12) becomes

$$\gamma_i = \sum_{j=1}^{n} \frac{\partial f_i(0, 0, \dots, 0)}{\partial \gamma_j}\gamma_j + \delta_i, \quad i = 1, 2, \dots, n, \tag{3.13}$$

where, upon a trivial change of variables $\lambda_{kj} = \partial f_i(0, 0, \dots, 0)/\partial \gamma_j$, (3.13) is perfectly analogous to our model in (3.4). It is worth mentioning here that the combined system of (3.8) through (3.13) will occasionally not have a closed-form solution. In those unique instances, we shall solve the alternative Leontief-type equilibrium (Section 3.3, along with later numerical examples, illustrates how this solution is attained):

$$\gamma_i = \min\{f_i(\gamma_1, \gamma_2, \dots, \gamma_n) + \delta_i, 1\}. \tag{3.14}$$

3.3 The dynamics of the model

We now examine how our system, as captured in (3.6), reaches its equilibrium state, ie, given some instantaneous initial perturbation in any of the triggering economic shocks, how would γ evolve over time? Further, we shall investigate whether this steady state is indeed exclusive. To simultaneously address these two inquiries, we ought to transform the problem at hand into a dynamic system that inspects gradual changes over time t in the vector $\gamma = [\gamma_1, \gamma_2, \dots, \gamma_n]^{\mathrm{T}}$, as follows:

$$\frac{\partial \gamma}{\partial t}(t) = f\left(\gamma, \frac{\partial \gamma}{\partial t}, t\right) + \psi(t), \tag{3.15}$$

where $f(\cdot)$ is some continuous, sufficiently differentiable nondecreasing function, having an initial condition $f(0) = 0$ (as discussed earlier); all state variables are strictly bounded to the domain of $[0, 1]$; and $\psi(t) = [\psi_1(t), \psi_2(t), \dots, \psi_n(t)]^{\mathrm{T}}$ defines the paces at which the initial perturbations in the set of macroeconomic shocks escalate over time. This stress test dynamic system has at least one convergence point, as captured by the Leontief-type equilibrium equation, where the absorbing state $\Delta(\tau^*) = 1$ exemplifies a regulatory bank failure as long as $\psi_i(t) \geqslant 0$.

Since our primal model is linear in nature, it is both intuitive and mathematically tractable to portray this positive dynamic process along linear equations as

$$\frac{\partial \gamma_i}{\partial t}(t) = \sum_j a_{ij}\gamma_j + \sum_j b_{ij}\frac{\partial \gamma_j}{\partial t} + \psi_i(t), \quad i = 1, 2, \dots, n. \tag{3.16}$$

In matrix notation, this dynamic system looks like

$$\frac{\partial \gamma}{\partial t}(t) = A\gamma + B\frac{\partial \gamma}{\partial t} + \psi(t), \tag{3.17}$$

where both A and B are $n \times n$ square matrixes, and

$$\psi(t) = [\psi_1(t), \psi_2(t), \dots, \psi_n(t)]^{\mathrm{T}}$$

is a column vector. When no perturbations take place, all economic shocks are null; thus, $\psi_i(t) = 0$ for all $i = 1, 2, \dots, n$. In this case, we can simplify (3.17) and obtain

$$\frac{\partial \gamma}{\partial t}(t) = A\gamma + B\frac{\partial \gamma}{\partial t}. \tag{3.18}$$

We recall that $\gamma(0) = 0$ and, therefore,

$$\frac{\partial \gamma}{\partial t}(0) = 0.$$

Under these (both necessary and sufficient) conditions, the existence and uniqueness theorem for a first-order differential equation dictates that (3.18) has an exclusive (unique) solution.[13] Because $\gamma = 0$ is in fact a trivial solution, it must also be the

[13] This theorem is also known in the mathematical literature as the Cauchy–Lipschitz theorem or the Picard–Lindelöf theorem. The interested reader can learn more about this in Coddington and Levinson (1955).

only solution. This proves that, with no economic shocks injected into the supervisory stress test, the examined BHC remains in its operative state, having fixed excess capital beyond the minimum regulatory threshold. However, with the introduction of an instantaneous economic tremor to the supervisory stress test, in the baseline, adverse or severely adverse scenarios this dynamic system would converge to a different equilibrium state.

3.4 Possible regulatory intervention

Regulators are mandated to interfere whenever they sense that BHCs in general and systemically important financial institutions (SIFIs) in particular might be in distress, a situation that can lead to "bank runs" and other liquidity problems. As mentioned earlier, in light of the Federal Reserve's CCAR, this regulatory intervention takes place to prevent any harm befalling either borrowers or lenders in the financial system; thus, the results of the supervisory stress tests could be substantially different pre-CCAR and post-CCAR. Nonetheless, while some regulatory interventions are done in advance, ie, before economic shocks strike, at other times these rescue actions are employed during economic crises. Obviously, preventative regulatory measures are always preferable; however, in practice, the Federal Reserve may launch emergency arrangements at any given time. In addition, these regulatory interventions can be deployed once (eg, one-time capital injections to distressed BHCs), but they can also be spread out over some period of time (eg, a regulatory interference in the dividend payout policy henceforth of a specific bank).

Suppose that a regulatory intervention can salvage $\mu_i(t)$ from the projected monetary damage associated with economic shock i to a BHC's excess capital beyond the minimum legal threshold. Similar to $\psi_i(t)$, $\mu_i(t) \in [0, 1]$, and – to enhance intuition – both are generally expressed in percentage amounts (after scaling both by the BHC's buffer zone). In this extended setting, the proposed framework simply becomes

$$\frac{\partial \gamma}{\partial t}(t) = A\gamma + B\frac{\partial \gamma}{\partial t} + \psi(t) - \mu(t), \tag{3.19}$$

where $\mu(t) = [\mu_1(t), \mu_2(t), \dots, \mu_n(t)]^{\mathrm{T}}$ is a transposed row vector (ie, a column vector) having the same dimensions of $n \times 1$ as $\psi(t)$.

The above formulas depict diverse economic settings that together form a comprehensive framework for regulatory stress tests with interconnected economic shocks and possible regulatory intervention. We now turn to demonstrate these venues with fictional yet representative numerical examples.

4 NUMERICAL EXAMPLES

In practice, whenever regulators and policy makers need to assess the mutual dependencies λ_{kj} between pairs of economic shocks j and k, they would typically rely on past dependencies, although further calibrations based on impulse responses should be appointed to shape matrix $\mathcal{S}$ in (3.5) and (3.6) as a more forward-looking dependency template. This technique, along with the necessary adjustments to better reflect feasible economic shocks during the future planning horizon, can serve as the groundwork that forms the 28×28 square dependency matrix $\mathcal{S}$, which contains zeros along its main diagonal.[14] We believe, however, that constructing the complete dependency matrix $\mathcal{S}$ here and then deploying our derivations thus far over this highly complex set of information would serve little purpose. In that state, computations would require the use of computers, which would fail to support readers' intuition. We therefore illustrate the functionality of our proposed framework with the following simple, universal and somewhat more intuitive examples.

4.1 Example #1

Let us assume that a supervisory stress test consists of only two feasible economic shocks. These macroeconomic shocks are time-lagged; hence, their respective degrees of dependency are not necessarily identical, ie, $\lambda_{1_t 2_{t-1}} \neq \lambda_{2_t 1_{t-1}}$. In particular, from past data (with or without modifications, which are based on some forward-looking projections), we learn that the 2×2 dependency matrix is

$$\mathcal{S} = \begin{pmatrix} 0 & 0.7 \\ 0.4 & 0 \end{pmatrix}.$$

This means that when the second economic shock strikes, it also triggers 0.7 of the magnitude of the first economic shock; however, when the first economic shock hits, it prompts only 0.4 of the magnitude of the second economic shock. Now, let us consider that a particular stress test gauges a $\delta_2 = 0.2$ magnitude, ie, the simulated in-isolation percentage monetary damage from the second round of macroeconomic turbulence to the BHC's excess capital beyond the minimum regulatory threshold is 20%. Under these circumstances, we can solve the stress test system

$$\begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} = \begin{pmatrix} 0 & 0.7 \\ 0.4 & 0 \end{pmatrix} \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0.2 \end{pmatrix} = \begin{pmatrix} 0.7\gamma_2 \\ 0.4\gamma_1 + 0.2 \end{pmatrix}.$$

This plain system of two equations and two unknowns yields a definite solution of $\{\gamma_1 \cong 0.194, \gamma_2 \cong 0.278\}$. This means that a solitary 20% magnitude perturbation within the second economic shock in fact instigates further losses to the BHC,

[14] We recall that $\lambda_{kk} = 0$ for all k along the main diagonal of matrix $\mathcal{S}$, and this is the reason for adding the separate term δ_k in (3.3) and (3.4).

not only because of the interconnectedness with the first economic shock but also because of the bounce-back effect on the second economic shock, which increases its own damage from 20% to about 27.8% of the BHC's excess capital. In this case, the total (accumulated) capital loss of the BHC's buffer zone is much greater than originally assumed by the in-isolation macroeconomic shock, since

$$\Delta = \gamma_1 + \gamma_2 \cong 0.194 + 0.278 = 0.472 = 47.2\%.$$

4.2 Example #2

For the setting described above (as well as for other similar systems), we can also form a comprehensive sensitivity analysis with varying intensity. We therefore study the general economic turbulence $\delta_2 = \eta \times 100\%$, meaning that $\eta \times 100\%$ magnitude is now the supervisory stress test's input, injected as the second economic tremor.[15] Consequently, the stress test system becomes

$$\begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} = \begin{pmatrix} 0 & 0.7 \\ 0.4 & 0 \end{pmatrix} \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \eta \end{pmatrix} = \begin{pmatrix} 0.7\gamma_2 \\ 0.4\gamma_1 + \eta \end{pmatrix},$$

which straightforwardly yields the broad solution of $\{\gamma_1 \cong 0.972\eta, \gamma_2 \cong 1.389\eta\}$; therefore, the total capital loss to the BHC's buffer zone is

$$\Delta = \gamma_1 + \gamma_2 \cong (0.972 + 1.389)\eta = 2.361\eta.$$

We therefore learn that in this particular supervisory stress test, the examined BHC fails when its excess capital beyond the minimum regulatory threshold is completely eradicated: hence, when $\Delta \geqslant 1$ or when $\eta \geqslant \frac{1}{2.361} = 0.424$. In essence, every perturbation bigger than 42.4% in the second macroeconomic shock would, by itself, instigate a supervisory bank failure, as illustrated in Figure 1. Following these lines, we can further replicate the sensitivity analysis for the first economic shock, or detect specific degrees of economic shocks that would cause varying damage to the examined BHC's excess capital beyond the minimum required by law.

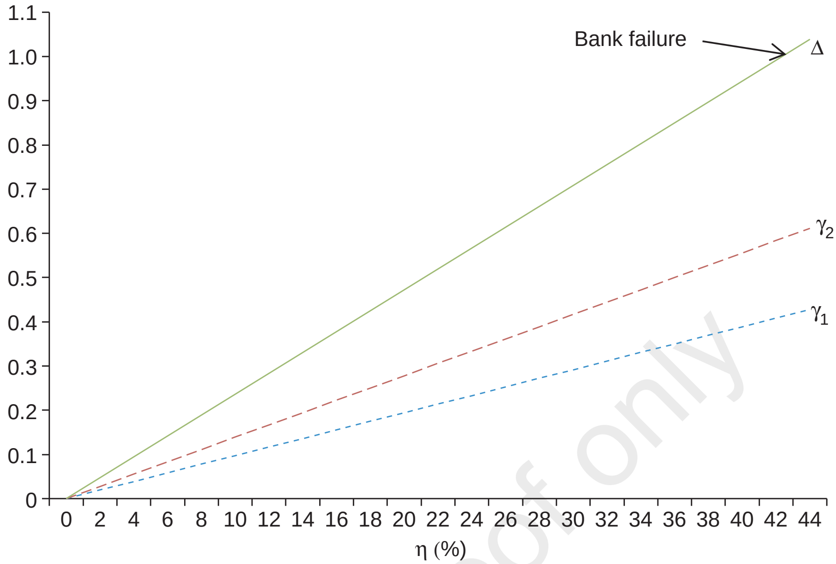
4.3 Example #3

Let us now consider a somewhat naïve stress test that contains two timely synchronized, perfectly connected macroeconomic shocks. In this case, the 2×2 dependency matrix $\mathcal{S}$ is

$$\mathcal{S} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

[15] This particular analysis takes a reversed approach somewhat similar to Kopeliovich *et al* (2015).

FIGURE 1 Sensitivity analysis for numerical examples #1 and #2.



In this simulation, we alternate $\eta \times 100\%$ as the supervisory stress test's input, injected into the second economic tremor, and measure its combined effects on the simultaneous equations $\gamma_1 = 0.7\gamma_2$ and $\gamma_2 = 0.4\gamma_1 + \eta$. We record the outcome as $\gamma_1 \cong 0.972\eta$ for the first economic shock, $\gamma_2 \cong 1.389\eta$ for the second economic shock and $\Delta = \gamma_1 + \gamma_2 \cong (0.972 + 1.389)\eta = 2.361\eta$ for the inclusive damage that the examined BHC suffers in its excess capital beyond the minimum regulatory level. A level of $\Delta \geqslant 1$ indicates a supervisory bank failure since the simulated bank's capital breaches the regulatory minimum threshold (the excess capital is completely eradicated).

Suppose also that the supervisory stress test would like to alternate the second macroeconomic variable and to introduce an instantaneous input shock with a magnitude of $\delta_2 = 0.5$. Subsequently, our stress test model reads

$$\begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 0.5 \end{pmatrix} = \begin{pmatrix} \gamma_2 \\ \gamma_1 + 0.5 \end{pmatrix}.$$

This system has no arithmetic solution that simultaneously satisfies $\gamma_1 = \gamma_2 = \gamma_1 + 0.5$. Thus, we seek a substitute resolution within the alternative equilibrium in (3.14). The trivial outcome of the alternative system $\gamma_1 = \min\{\gamma_2, 1\}$ and $\gamma_2 = \min\{\gamma_1 + 0.5, 1\}$ is $\gamma_1 = \gamma_2 = 1$. This means that the tested bank is destined to fail at some point in the future. We therefore would like to learn more about the dynamic process that will lead to this inevitable aftermath. From quarterly (arbitrarily selected time units) actuarial data, we deploy standard curve-fitting techniques and estimate the dynamic processes that shift the two underlying economic variables, so the matrixes A and B in (3.17) are, respectively,

$$A = \begin{pmatrix} 0 & 0.4 \\ 0.2 & 0 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

To truthfully describe an instantaneous tremor at a magnitude of $\delta_2 = 0.5$ only in the second economic variable, we further shape (3.17) with $\psi(t) = [0, 0.5\delta(t)]^{\mathrm{T}}$, where

$$\delta(t) = \begin{cases} 1 & \text{for } t = 0, \\ 0 & \text{for } t \neq 0, \end{cases}$$

ie, the shock injected into the second macroeconomic variable in this stress test attains half a Dirac delta function. We can now solve this dynamic setting as described hereafter. The BHC's future capital losses progress along two unparalleled forces:

$$\left\{ \frac{\partial \gamma_1}{\partial t} = 0.4\gamma_2; \ \frac{\partial \gamma_2}{\partial t} = 0.2\gamma_1 + 0.5\delta(t) \right\}.$$

Immediately (with an infinitesimal time increment) after the economic shock is implanted into the second macroeconomic variable, the initial conditions dictate $\gamma_1(0^+) = 0$ and $\gamma_2(0^+) = 0.5$. In addition, by taking the second derivative of γ_1 any time thereafter (so $\delta(t) = 0$), we get

$$\frac{\partial^2 \gamma_1}{\partial t^2} = 0.4 \frac{\partial \gamma_2}{\partial t} = 0.4 \times 0.2\gamma_1 = 0.08\gamma_1,$$

which eventually yields the final result:

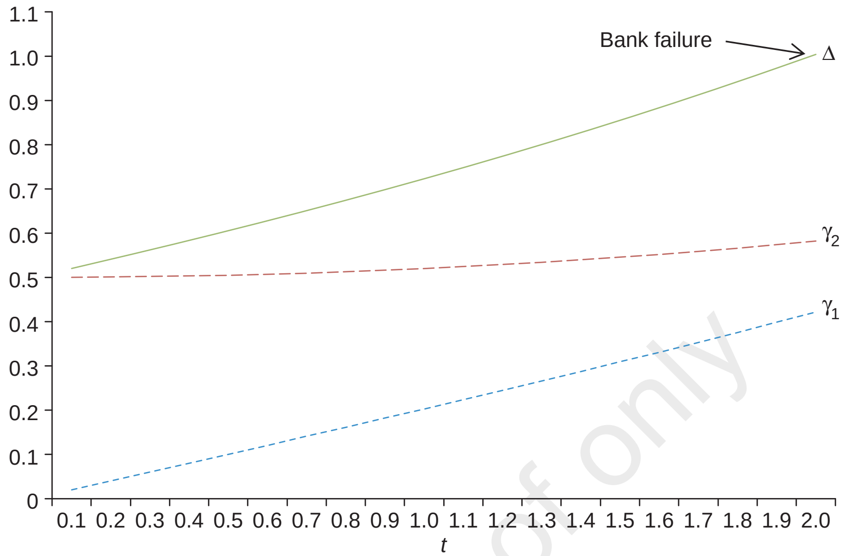
$$\gamma_1 = \frac{\mathrm{e}^{\sqrt{2}t/5} - \mathrm{e}^{-\sqrt{2}t/5}}{2\sqrt{2}}, \qquad \gamma_2 = \frac{\mathrm{e}^{\sqrt{2}t/5} + \mathrm{e}^{-\sqrt{2}t/5}}{4},$$

$$\Delta = \gamma_1 + \gamma_2 = \frac{(1 + \sqrt{2})\mathrm{e}^{\sqrt{2}t/5} + (1 - \sqrt{2})\mathrm{e}^{-\sqrt{2}t/5}}{4},$$

where $\mathrm{e} \cong 2.718$ denotes the base of the natural logarithm. This means that the examined BHC is doomed to fail this supervisory stress test after about two quarters of a year (when $t = 2$), since then $\Delta \cong 1.003$, which completely eradicates the bank's excess capital beyond the minimum regulatory threshold. We illustrate the entire evolution of this ergodic bank failure in Figure 2.[16]

[16] Interested readers should note the convexity of the paths of both γ_2 and Δ along time. This common phenomenon implies that the monetary damage to the BHC increases exponentially in light of the mutual dependency between the macroeconomic variables in this supervisory stress test.

FIGURE 2 The dynamic process for numerical Example #3.



In this simulation, we infuse at time $t = 0$ an instantaneous input shock with a magnitude of $\delta_2 = 0.5$ into the second macroeconomic variable. Given the perfect dependency assumed between the two economic variables in this particular bank stress test, and without any regulatory intervention, the examined BHC is doomed to fail. To closely monitor the bank's path to failure, we track how γ_1 as the bank's capital erosion associated with the first economic variable, γ_2 as the bank's capital destruction associated with the second economic variable and $\Delta = \gamma_1 + \gamma_2$ as the accumulated capital loss all progress over time. A level of $\Delta \geqslant 1$ indicates a supervisory bank failure, since the simulated bank's capital breaches the regulatory minimum threshold (the excess capital is completely eliminated).

4.4 Example #4

Following Example #3, let us assume that regulatory intervention is designed to take place to address the harmful shock in the second economic variable. This backup arrangement, however, is not deployed in advance. Instead, it is planned to be installed by the Federal Reserve only after one quarter of a year, ie, at $t = 1$, merely upon some financial deterioration, and it is meant to continue thereafter at least until the BHC stabilizes with a rate of 0.2. Explicitly, it should restore 20% of the BHC's excess capital above the minimum regulatory level in every consecutive quarter henceforth.

In this extended scenario, the stress test model behaves as in Example #3 during the first quarter, ie, for $0^+ \leqslant t < 1$. However, at exactly $t = 1$ the regulatory intervention starts to influence the system, and from this point on it continues with the following differential equations:

$$\left\{\frac{\partial\gamma_1}{\partial t}=0.4\gamma_2;\ \frac{\partial\gamma_2}{\partial t}=0.2\gamma_1-0.2\right\},$$

subject to the initial conditions (at $t=1$)

$$\gamma_1(1)=\frac{\mathrm{e}^{\sqrt{2}/5}-\mathrm{e}^{-\sqrt{2}/5}}{2\sqrt{2}}\cong 0.203,\qquad \gamma_2(1)=\frac{\mathrm{e}^{\sqrt{2}/5}+\mathrm{e}^{-\sqrt{2}/5}}{4}\cong 0.520,$$

$$\Delta(1)=\gamma_1(1)+\gamma_2(1)\cong 0.723.$$

Note that after one quarter (at $t=1$), the deterioration in the BHC's excess capital is rather significant, since 72.3% of the buffer zone is missing by now. We can therefore examine whether the undertaken regulatory intervention is sufficient (both in terms of magnitude and reaction time) to save the examined bank. Since we already discovered that the system trails

$$\frac{\partial\gamma_1}{\partial t}=\frac{2}{5}\gamma_2,$$

this means that

$$\frac{\partial\gamma_2}{\partial t}=\frac{5}{2}\frac{\partial^2\gamma_1}{\partial t^2},$$

and because

$$\frac{\partial\gamma_2}{\partial t}=0.2\gamma_1-0.2,$$

we obtain

$$\frac{5}{2}\frac{\partial^2\gamma_1}{\partial t^2}=\frac{\gamma_1-1}{5}.$$

The solution for this equality takes the general form

$$\gamma_1=1+C_1\mathrm{e}^{\sqrt{2}t/5}+C_2\mathrm{e}^{-\sqrt{2}t/5},$$

where C_1 and C_2 are some constants. Once we utilize the initial (at $t=1$) conditions listed above, we get $C_1\cong-0.02326$ and $C_2\cong-1.01699$; thus, the system converges to

$$\gamma_1=1-0.02326\mathrm{e}^{\sqrt{2}t/5}-1.01699\mathrm{e}^{-\sqrt{2}t/5},$$

$$\gamma_2=-0.01645\mathrm{e}^{\sqrt{2}t/5}+0.7191\mathrm{e}^{-\sqrt{2}t/5},$$

$$\Delta=\gamma_1+\gamma_2=1-0.03971\mathrm{e}^{\sqrt{2}t/5}-0.29789\mathrm{e}^{-\sqrt{2}t/5}.$$

It appears that the regulatory intervention within this stress test extended system is able to rescue the examined BHC just in time, as illustrated in Figure 3. This regulatory intervention has a rate of 0.2 per quarter; thus, it is able to reverse the damage already done by the second economic shock in a way that, starting from $t = 1$, the monetary loss associated with the second economic shock γ_2 gradually reduces. In light of this external intervention, the impact of the second economic shock completely dissolves (reduces to less than 0.5%) after $t = 6.6$ quarters of a year. Simultaneously, the capital loss linked to the first economic shock γ_1 keeps on increasing, yet at a slower pace than before in light of the positive relation between the two macroeconomic shocks. Consequently, the accumulated capital loss to the BHC Δ reaches its highest level of nearly 78.25% after $t = 3.6$ quarters (almost a year), and then it starts to decay.[17] In practice, however, the Federal Reserve would preferably react sooner and more aggressively to alleviate the harmful effect of both economic shocks.

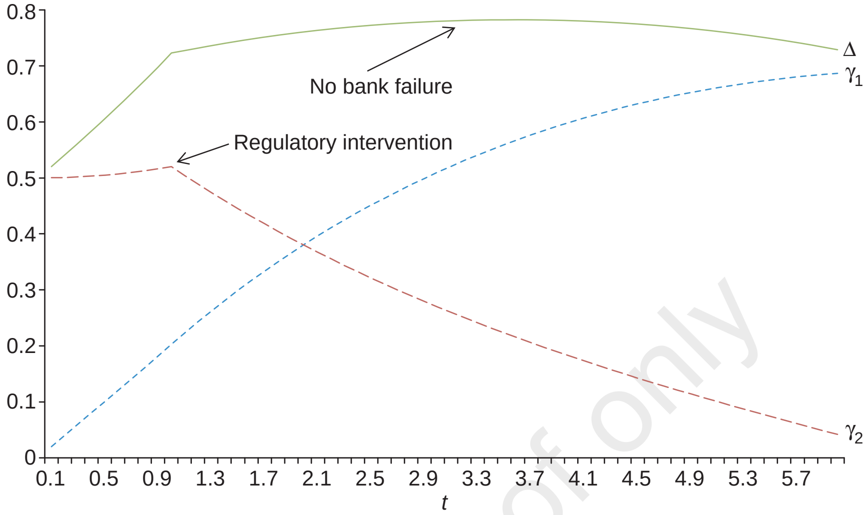
5 SUMMARY

In this study, we have developed, justified and demonstrated a generic framework for supervisory bank stress tests to be used by regulatory bodies. The proposed differential equations model accentuates the mutual dependencies among macroeconomic shocks normally observed (but which can also be modified or projected). As such, our model computes more accurately the aggregated damage expected to a BHC's excess capital beyond the minimum regulatory threshold. The enhanced flexibility of the present framework permits supervisors to explore a large (and, in fact, continuous) range of economic scenarios and not necessarily follow the three basic settings commonly used (baseline, adverse and severely adverse settings). The current framework further allows for possible regulatory intervention to be embedded within the broader CCAR program. It thus assists policy makers with examining in advance the likely impact of various preventative measures and emergency rescue actions along different paths and at diverse points in time. It can therefore help regulators to select the optimal intervention (if required) in numerous economic circumstances.

As future lines of research, we recommend that intrigued scholars try to assimilate the current framework with stochastic macroeconomic shocks $\delta_k \in [0, 1]$ and perhaps dynamic dependencies $\lambda_{kj} \in [0, 1]$. The resulting simulated paths would likely track multiple instantaneous equilibrium points along the regulatory planning horizon. The entire collection of these stochastic paths (upon running numerous Monte Carlo simulations) may portray a clearer image of the real chances for BHCs to survive a highly volatile economic environment. Alternatively, specific distributional properties (based on past experience) can be further assigned to the parameters of our

[17] All of the above projected turning events and their respective algebraic inflection points are perfectly situated within the standard Federal Reserve's planning horizon.

FIGURE 3 The dynamic process with the regulatory intervention in numerical Example #4.



In this simulation, we extend Example #3 and further introduce regulatory intervention (devised during the CCAR program as an emergency rescue action that should counter a shock in the second economic variable) after one quarter of a year, ie, at $t = 1$, with a capacity to restore 20% of the BHC's excess capital above the minimum regulatory level in every consecutive quarter henceforth. In this case, the stress test model behaves exactly as in Example #3 during the first quarter but changes its course immediately after. The planned regulatory intervention is able to gradually decrease the harmful effect of the second economic variable γ_2 until it effectively vanishes after $t = 6.6$ quarters of a year. At the same time, the capital loss associated with the first economic variable γ_1 keeps on increasing, but at a slower pace than before. Consequently, the accumulated capital loss to the BHC $\Delta = \gamma_1 + \gamma_2$ reaches its highest level of nearly 78.25% after $t = 3.6$ quarters (almost a year) before starting to decay; hence, the BHC is salvaged on time.

model. These potential research avenues add some complexity to the present framework. Nonetheless, given the advanced computational power most software tools exhibit these days, these supplements are feasible and rather informative.

DECLARATION OF INTEREST

The authors report no conflicts of interest. The authors alone are responsible for the content and writing of the paper.

ACKNOWLEDGEMENTS

The authors wish to thank the anonymous referee and Patricia Nickinson for helpful editorial comments.

REFERENCES

Alexander, C., and Sheedy, E. (2008). Developing a stress testing framework based on market risk models. *Journal of Banking and Finance* **32**, 2220–2236 (https://doi.org/10.1016/j.jbankfin.2007.12.041).

Aragones, J. R., Blanco, C., and Dowd, K. (2001). Incorporating stress tests into market risk modeling. *Derivatives Quarterly*, Spring, 44–49.

Bangia, A., Diebold, F. X., Kronimus, A., Schagen, C., and Schuermann, T. (2002). Ratings migration and the business cycle, with application to credit portfolio stress testing. *Journal of Banking and Finance* **26**, 445–474 (https://doi.org/10.1016/S0378-4266(01)00229-1).

Berkowitz, J. (1999). A coherent framework for stress-testing. *The Journal of Risk* **2**(2), 5–15 (https://doi.org/10.21314/JOR.2000.021).

Board of Governors of the Federal Reserve System (2016a). Dodd–Frank Act stress test 2016: supervisory stress test methodology and results. Report, June. URL: https://www.federalreserve.gov/newsevents/pressreleases/files/bcreg20160623a1.pdf.

Board of Governors of the Federal Reserve System (2016b). Comprehensive Capital Analysis and Review 2016: assessment framework and results. Report, June. URL: https://www.federalreserve.gov/newsevents/pressreleases/files/bcreg20160629a1.pdf.

Breeden, J. L., Thomas, L., and McDonald, J. W. (2008). Stress-testing retail loan portfolios with dual-time dynamics. *The Journal of Risk Model Validation* **2**, 43–62 (https://doi.org/10.21314/JRMV.2008.033).

Čihák, M. (2007). Introduction to applied stress testing. Working Paper 59, March, International Monetary Fund (https://doi.org/10.5089/9781451866230.001).

Coddington, E. A., and Levinson, N. (1955). *Theory of Ordinary Differential Equations*. McGraw-Hill, New York.

Dimson, E., and Marsh, P. (1997). Stress tests of capital requirements. *Journal of Banking and Finance* **21**, 1515–1546 (https://doi.org/10.1016/S0378-4266(97)00047-2).

Drehmann, M., Sorensen, S., and Stringa, M. (2010). The integrated impact of credit and interest risk on banks: a dynamic framework and stress testing application. *Journal of Banking and Finance* **34**, 713–729 (https://doi.org/10.1016/j.jbankfin.2009.06.009).

Fender, I., Gibson, M. S., and Mosser, P. C. (2001). An international survey of stress tests. *Federal Reserve Bank of New York Current Issues in Economics and Finance* **7**(10), 1–6 (https://doi.org/10.2139/ssrn.711382)

Foglia, A. (2009). Stress testing credit risk: a survey of authorities' approaches. *International Journal of Central Banking* **5**(3), 9–37 (https://doi.org/10.2139/ssrn.1396243).

Goodhart, C. A. E. (2006). A framework for assessing financial stability? *Journal of Banking and Finance* **30**(12), 3415–3422 (https://doi.org/10.1016/j.jbankfin.2006.06.003).

Goodhart, C. A. E., Sunirand, P., and Tsomocos, D. (2004). A model to analyze financial fragility: applications. *Journal of Financial Stability* **1**(1), 1–30 (https://doi.org/10.1016/j.jfs.2004.06.005).

Goodhart, C. A.E., Sunirand, P., and Tsomocos, D. (2006). A model to analyze financial fragility. *Economic Theory* **27**(1), 107–142 (https://doi.org/10.1007/s00199-004-0572-7).

Greenspan, A. (2000). Greenspan's plea for stress-testing. *Risk* **3**(6), 53–55.

Horowitz, K. J., and Planting, M. A. (2006). Concepts and methods of the US input–output accounts. Handbook, September, Bureau of Economic Analysis, Suitland, MD. URL: https://www.bea.gov/resources/methodologies/concepts-methods-io-accounts.

Jackson, P., Maude, D. J., and Perraudin, W. (1997). Bank capital and value at risk. *Journal of Derivatives* **4**(3), 73–89 (https://doi.org/10.3905/jod.1997.407972).

Jacobs, M., Jr. (2013). Stress testing credit risk portfolios. *Journal of Financial Transformation* **37**, 53–76.

Jacobs, M., Jr., Karagozoglu, A. K., and Sensenbrenner, F. J. (2015). Stress testing and model validation: application of the Bayesian approach to a credit risk portfolio. *The Journal of Risk Model Validation* **9**(3), 41–70 (https://doi.org/10.21314/JRMV.2015.140).

Kearns, A. (2004). Loan losses and the macroeconomy: a framework for stress testing credit institutions' financial well-being. Financial Stability Report, pp. 111–121, Central Bank of Ireland.

Kida, M. (2008). A macro stress testing model with feedback effects. Discussion Paper DP2008/08, Federal Reserve Bank of New Zealand.

Kopeliovich, Y., Novosyolov, A., Satchkov, D., and Schachter, B. (2015). Robust risk estimation and hedging: a reverse stress testing approach. *Journal of Derivatives* **22**(4), 10–25 (https://doi.org/10.3905/jod.2015.2015.1.043).

Kupiec, P. H. (1998). Stress testing in a value at risk framework. *Journal of Derivatives* **6**(1), 7–24 (https://doi.org/10.3905/jod.1998.408008).

Kupiec, P. H. (1999). Risk capital and VaR. *Journal of Derivatives* **7**(2), 41–52 (https://doi.org/10.3905/jod.1999.319145).

Leontief, W. W. (1951). Input–output economics. *Scientific American* **185**(4), 15–21 (https://doi.org/10.1038/scientificamerican1051-15).

Longin, F. M. (2000). From value at risk to stress testing: the extreme value approach. *Journal of Banking and Finance* **4**, 1097–1130 (https://doi.org/10.1016/S0378-4266(99)00077-1).

Ramey, V. A. (2016). Macroeconomic shocks and their propagation. In *Handbook of Macroeconomics*, Taylor, J. B., and Uhlig, H. (eds), Volume 2, Chapter 2, pp. 71–162. Elsevier (https://doi.org/10.1016/bs.hesmac.2016.03.003).

Rebonato, R. (2010). *Coherent Stress Testing: A Bayesian Approach*. Wiley.

Rösch, D., and Scheule, H. (2007). Stress-testing credit risk parameters: an application to retail loan portfolios. *The Journal of Risk Model Validation* **1**(1), 55–75 (https://doi.org/10.21314/JRMV.2007.004).

Sharman, D. M., and Yassine, A. A. (2004). Characterizing complex product architectures. *Systems Engineering* **7**(1), 35–60 (https://doi.org/10.1002/sys.10056).

Sorge, M. (2004). Stress-testing financial systems: an overview of current methodologies. Working Paper 165, Bank for International Settlements (https://doi.org/10.2139/ssrn.759585).

Sorge, M., and Virolainen, K. (2006). A comparative analysis of macro stress-testing methodologies with application to Finland. *Journal of Financial Stability* **2**, 113–151 (https://doi.org/10.1016/j.jfs.2005.07.002).

Tan, K. H., and Chan, I. L. (2003). Stress testing using VaR approach: a case for Asian currencies. *International Financial Markets, Institutions and Money* **13**, 39–55 (https://doi.org/10.1016/S1042-4431(02)00025-2).

Wong, J. H. Y., Choi, K. F., and Fong, P. W. (2008). A framework for stress-testing banks' credit risk. *The Journal of Risk Model Validation* **2**(1), 3–23 (https://doi.org/10.21314/JRMV.2008.018).