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Analyzing the Long-Term Performance of the Defaulted Debt Market: Implications for Investors and Risk Managers

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Abstract - **This study empirically analyzes the historical performance of defaulted debt from Moody's (1987-2010). Motivated by a stylized structural model of credit risk with systematic recovery risk, we argue and find evidence that returns on defaulted debt covary with determinants of the market risk premium, firm specific and structural factors. Returns increase (decrease) in issue collateral quality or debt cushion; for issuers having higher ratings, leverage, higher Cumulative Abnormal Returns on equity or market implied loss severity; and with equity market indices (industry default rates or short-term interest rates.) This is relevant for investors and risk managers in this segment.**

Keywords: **Distressed debt; Recoveries; Default; Credit risk**
JEL Codes: **G33, G34, C25, C15, C52**

1. Introduction and Summary

There exists an economic argument that to the extent there may be opportunity costs associated with holding defaulted debt, and that the performance of such debt may vary systematically, the required return on the defaulted instruments should include an appropriate risk premium. Thus far, most research studying systematic variation in defaulted debt recoveries has focused on the influence of either macroeconomic factors ((Cary and Gordy (2007), Frye (2000 a,b,c; 2003), Hu and Perraudin (2002)) or supply / demand conditions in the defaulted debt markets (Altman et al, 2003), as determinants of recoveries (e.g., collateral values) are thought co-vary with such systematic macroeconomic measures. This previous research has found mixed empirical results regarding systematic variation in recoveries, which we argue is due to the unmeasured factors influencing the market risk premium for defaulted debt. Adequately controlling for other determinants of defaulted debt performance, potentially imperfectly correlated with standard macroeconomic indicators, is critical to understanding this.

We propose to extend this literature in several ways. First, we quantify the systematic variation in defaulted debt returns with respect to factors which influence the market risk premium for defaulted debt, which are related to investors' risk aversion or investment opportunity sets[1]; in the process, we specify a simple stylized model of credit risk in structural framework (Merton, 1974), having testable implications that are investigated herein. Second, we are able to analyze defaulted debt performance in segments homogenous with respect to recovery risk, through controlling for both for firm and instrument specific covariates, and examine whether it is associated with recoveries on defaulted debt securities. Third, departing from most of the prior literature on recoveries, having either focused on measures either at around the time of default or at settlement, through studying the relationship amongst these in the form of returns. We believe that such focus is most relevant to market participants – either traders or buy-and-hold investors (e.g., vulture funds, or financial institutions managing defaulted portfolios) – since in finance this is an accepted measure of economic gain or loss. Fourth, we are able to build parsimonious and robust econometric models, in the *generalized linear model* (GLM) class, that are capable to explaining and predicting defaulted debt returns.

In this study, we quantify the performance of defaulted debt relative to the previously and newly proposed determinants of corporate debt recoveries, through a comprehensive analysis of the returns on this

[1] Standard portfolio separation theory implies that, all else equal, during episodes of augmented investor risk aversion, a greater proportion of wealth is allocated to risk-free assets (Tobin (1958), Merton (1971)), implying lessened demand and diminished expected returns across all risky assets.

asset class. The data-set that we utilize, Moody's *Ultimate Recovery Database*™ (MURD™), contains the market prices of defaulted bonds and loans near the time of default, and the prices of these instruments (or market value of the bundle of instruments received in settlement of default) at the resolution of default. We have such data for 550 obligors and 1368 bonds and loans in the period 1987-2009. We examine the distributional properties of the individual annualized rates of return on defaulted debt across different segmentations in the dataset (e.g., default type, facility type, time period, seniority, collateral, original rating, industry), and build econometric models to explain observed returns.

Our principle results are as follows. We find returns to be in line with (albeit to the upper end of the range of results) what has been found in the previous literature, a mean of 28.6%[2]. We find returns on defaulted debt to vary significantly according to contractual, obligor, equity / debt market and economic factors. At the facility structure level, there is some evidence that returns are elevated for defaulted debt having better collateral quality rank or better protected tranches within the capital structure. At the obligor or firm level, returns are elevated for obligors rated higher at origination, more financially levered at default or having higher Cumulative Abnormal Returns (CARs) on equity prior to default. However, we also find returns to be increasing in the market implied loss severity at default. We also find evidence that defaulted debt returns vary pro-cyclically, as they decrease with industry default rates, and increase with aggregate equity market returns. Further, we observe that short- term interest rates are inversely related to returns on defaulted debt.

In addition to the relevance of this research for resolving questions in the finance of distressed debt investing, and aiding practitioners in this space, our results have implications for recently implemented supervisory Basel II capital standards for financial institutions (BCBS, 2004). Our results indicate that time variation in the market risk premium for defaulted debt may be an important systematic factor influencing recoveries on such instruments (and by implication, their *loss-given-default* – LGD), which is likely to not be perfectly correlated with the business cycle. Hence, any financial institution, in making the decision about how much capital to hold as a safeguard against losses on corporate debt securities, should take into account factors such as the systematic variation in investor risk aversion and investment opportunity sets[3]. Indeed, Basel II requires that banks quantify "downturn effects" in LGD estimation (BCBS, 2005), and for the relevant kind of portfolio (i.e., large corporates having marketable debt), and our research provides some guidance in this regard.

This study will proceed as follows. Section 2 reviews the relevant literature. Section 3 discusses the theoretical framework. Section 4 presents our empirical methodology. Section 5 summarizes returns according to various segmentations of the data. Section 6 presents summary statistics of our available covariates and Section 7 presents results of multiple regression analysis of defaulted debt returns. Section 8 concludes and provides possible directions for future research.

2. Review of the Related Literature

Altman (1989) develops a then new methodology for the measurement of risk due to default, suggesting a means of ranking fixed-income performance over a range of credit-quality segments. This technique measures the expected mortality of bonds, and associated loss rates, similarly to actuarial tabulations that assess human mortality. Results demonstrate outperformance by risky bonds relative to riskless Treasuries over a ten-year horizon and that, despite relatively high mortality rates, B-rated and CCC-rated securities outperform all other rating categories in the first four years after issuance, with BB-rated securities outperforming all others thereafter.

Gilson (1995) surveys the market practices of so-called "vulture investors", noting that as the risks of such an investment style exposes one to a high level of idiosyncratic and non-diversifiable risk, those who succeed in thus space must have a mastery of legal rules that govern corporate bankruptcy. The author further argues that a mastery of the legal and institutional setting can result in very high returns. Hotchkiss and Mooradian (1997) study the function of this investor class in the governance and reorganization of defaulted firms using a sample of 288 public debt defaults. They attribute better relative operating performance after default to vulture investors gaining control of the target firm in either in a senior executive or an ownership role. They also find positive abnormal returns for the defaulted firm's equity

[2] The probable reason why we are closer to the higher end of estimates, such as Keenan et al (2000), is probably that we have included several downturn periods, such as the early 1990s and recently.

[3] Our research also has a bearing on the related and timely issue of the debate about the so-called "pro-cyclicality" of the Basel capital framework (Gordy, 2006), an especially relevant topic in the wake of the recent financial crisis, where a critique of the regulation is such that banks wind up setting aside more capital just at the time that they should be using capital to provide more credit to businesses or to increase their own liquidity positions, in order to help avoid further financial dislocations and help revitalize the economy.

or debt in the two days around public revelation of a vulture purchase of such instruments. The authors conclude that vulture investors add value by disciplining managers of distressed firms.

The historical performance of the Moody's Corporate Bond index (Keenan et al, 2000) shows annualized 17.4% in the period 1982-2000. However, this return has been extremely volatile, as most of this gain (147%) occurred in the period 1992-1996. Keenan et al (2000) and Altman and Jha (2003) both arrive at estimates of a correlation to the market on this defaulted loan index of about 20%, implying a market risk premium of 216 bps. Davydenko and Strebuleav (2002) report similar results for non-defaulted high-yield corporate bonds (BB rated) in the period 1994-1999.

In the perspective of viewing defaulted debt as an asset class, Guha (2003) documents a convergence in market value as a proportion of par with respect to bonds of equal priority in bankruptcy approaching default. This holds regardless of contractual features, such as contractual rate or remaining time-to-maturity. The implication is that while prior to default bonds are valued under uncertain timing of and recovery in the event of default, that varies across issues according to both borrower and instrument characteristics, upon default such expectations become one and the same for issues of the same ranking. There is cross-sectional variation in yields due to varied perceived default risk as well as instrument structures, but as default approaches the claim on the debt collapses to a common claim on the expected share of emergence value of the firm's assets due to the creditor class. Therefore, the contract rate on the debt pre-default is no longer the relevant valuation metric with respect to restructured assets. This was predicted by the Merton (1974) theoretical framework that credit spreads on a firm's debt approach the expected rate of return on the firm's assets, as leverage increases to the point when the creditors become the owners of the firm. Schuermann (2003) echoed the implications of this argument by claiming that cash flows post-default represent a new asset.

Altman and Jha (2003), regressing the Altman / Solomon Center defaulted bond index on the S&P 500 returns for the period 1986-2002, come up with an 11.1% required return (based upon a 20.3% correlation estimate). Altman et al. (2003) examine the determinants of recoveries on defaulted bonds, in a setting of systematic variation in aggregate recovery risk, based on market values of defaulted debt securities shortly following default. The authors find that the aggregate supply defaulted debt securities, which tends to increase in downturn periods, is a key determinant of aggregate as well as instrument level recovery rates. The authors' results suggest that while systematic macroeconomic performance may be associated with elevated LGD, the principle mechanism by which this operates is through supply and demand conditions in the distressed debt markets.

Machlachlan (2004), in the context of proposing an appropriate discount rate for workout recoveries for regulatory purposes in estimating economic LGD (BCBS, 2005), outlines a framework that is motivated by a single factor CAPM model and obtains similar results in two empirical exercises. First, regressing Altman-NYU Salomon Center Index of Defaulted Public Bonds in the period 1987-2002 on the S&P 500 equity index, a 20% correlation also obtains, implying a market risk premium of 216 bps. Second, he looks at monthly secondary market bid quotes for the period April 2002-August 2003, obtaining a beta estimate of 0.37, which according to the Frye (2000 c) extension of the Basel single factor framework, implies a recovery value correlation of 0.21 and an MRP of 224 bps.

Acharya et al (2007) examine the empirical determinants of ultimate LGD at the instrument level, and find that the association between the aggregate supply of defaulted debt securities and recoveries does not remain after controlling for industry level distress. They argue for a "fire-sale effect" that results when most firms in a troubled industry may be selling collateral at the same time. These authors' results imply that systematic macroeconomic performance may not be a sole or critical determinant of recovery rates on defaulted corporate debt. Carey and Gordy (2007) examine whether there is systematic variation in ultimate recoveries at the obligor (firm-level default incidence) level, and find only weak evidence of systematic variation in recoveries. We contribute to this literature by providing evidence regarding a previously unexamined systematic variation in recoveries on defaulted debt securities.

Most recently, Altman (2010) reports that the Altman-NYU Salomon Center Index of defaulted bonds (bank loans) returned 12.6% (3.4%) over the period 1986-2009 (1989-2009).

3. Theoretical Framework

In this section we lay out the theoretical basis for returns on post-default recoveries, denoted r_s^D, where s denotes a recovery segment (e.g., seniority classes, collateral types, etc.). Following an intertemporal version of the structural modelling framework for credit risk (Merton, 1971; Vasicek 1987, 2002)[4], we may write the stochastic

[4] Note that this is also the approach underlying the regulatory capital formulae (BCBS, 2004), as developed by Gordy (2003).

process describing the instantaneous evolution of the i^{th} firm's[5] asset return at time t as:

$$\frac{dV_{i,t}}{V_{i,t}} = \mu_i dt + \sigma_i dW_{i,t} \tag{3.1}$$

Where $V_{i,t}$ is the asset value, σ_i is the return volatility, μ_i is the drift (which can be taken to be the risk-free rate r under risk-neutral measure), and $W_{i,t}$ is a standard Weiner process that decomposes as (this is also known as a *standardized asset return*):

$$dW_{i,t} = \rho_{i,X} dX_t + \sqrt{1-\rho_{i,X}^2}\, dZ_{i,t} \tag{3.2}$$

Where the processes (also standard Weiners) X_t and $Z_{i,t}$ are the systematic risk factor (or standardized asset return) and the idiosyncratic (or firm-specific) risk factor, respectively; and the factor loading $\rho_{i,X}$ is constant across all firms in a *PD* segment homogenous with respect to default risk (or across time for the representative firm)[6]. It follows that the instantaneous asset-value correlation amongst firms (or segments) i and j is given by:

$$\frac{1}{dt} Cor_{i,j}^V \left[\frac{dV_{i,t}}{V_{i,t}}, \frac{dV_{j,t}}{V_{j,t}} \right] = \rho_{i,X} \rho_{j,X} \tag{3.3}$$

Defining the recovery rate on the i^{th} defaulted asset[7] at time t as $R_{i,t}$, we may similarly may write the stochastic process describing its evolution as:

$$\frac{dR_{i,t}}{R_{i,t}} = \mu_i^R dt + \sigma_i^R dW_{i,t}^R \tag{3.4}$$

Where μ_i^R is the drift (which can be taken to be the expected instantaneous return on collateral under physical measure, or the risk-free rate under risk-neutral measure), σ_i^R is the volatility of the collateral return and $W_{i,t}^R$ is a standard Weiner process that for recovery segment s^R decomposes as:

$$dW_{i,t}^R = \rho_{i,s^R} dX_t^R + \sqrt{1-\rho_{i,s^R}^2}\, dZ_{i,t}^R \tag{3.5}$$

Where the two-systematic factors are bivariate standard normal, each standard normal, but with correlation r between each other:

$$\left(dX_t, dX_t^R\right)^T \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & r \\ r & 1 \end{pmatrix} \right) \tag{3.6}$$

This set-up follows various extensions of the structural model framework for systematic recovery risk, which have in common that they allow the recovery process to depend upon a 2nd systematic factor, which may be correlated with the macro (or market) factor X_t (Frye (2000 a, b, c), Pyktin (2003), Dullman and Trapp (2004), Giese (2005), Rosch and Scheule (2005), Hillebrand (2006) and Barco (2007)). In this general and more realistic framework, returns on defaulted debt may be governed by a stochastic process distinct from that of the firm; this is the case where the asset is secured by cash, third party guarantees or assets not used in production. In this setting it is possible that there are two salient notions of asset value correlation, one driving the correlation amongst defaults, and another driving the correlation between collateral values and the returns on defaulted assets in equilibrium. This reasoning implies that it is entirely conceivable that, especially in complex banking facilities, cash flows associated with different sources of repayment should be discounted differentially according to their level of systematic risk. In not distinguishing how betas may differ between defaulted instruments secured differently, it is highly likely that investors in distressed debt may misprice such assets.

It is common to assume that the factor loading in (3.5) is constant amongst debt instruments within specified recovery segments, so that the recovery-value correlation for recovery a segment s^R is given by $\rho_{i,s^R}^2 \equiv R_{s^R}$ [8]. If we take the further step of identifying this correlation with the correlation to a market portfolio - arguably a reasonable interpretation in the *asymptotic single risk-factor* (ASRF) framework (Vasicek, 1987; Gordy, 2003) – then we can write $R_{s^R}^2 = \rho_{s^R,M}^2$. It then follows from the standard *capital asset pricing model* (CAPM) that the relationship between the firm and market rates of return is given by the beta coefficient:

[5] This could also be interpreted as the i^{th} *PD* segment or an obligor rating.

[6] Vasicek (2002) demonstrates that under the assumption of a single systematic factor, an infinitely granular credit portfolio and *LGD* that does not vary systematically, a closed-form solution for capital exists that is invariant to portfolio composition.

[7] We can interpret this as an *LGD* segment (or rating) or debt seniority class.

[8] Indeed, for many asset classes the Basel II framework mandates constant correlation parameters equally across all banks, regardless of particular portfolio exposure to industry or geography. However, for certain exposures, such as wholesale non-high volatility commercial real estate, this is allowed to depend upon the *PD* for the segment or rating (BIS, 2004).

$$\frac{Cov_{s^R,M}\left[\frac{dR_{i,t}}{V_{i,t}},\frac{dV_{M,t}}{V_{M,t}}\right]}{Var_M\left[\frac{dV_{M,t}}{V_{M,t}}\right]}=\beta_{s^R,M}=\frac{\sigma_i^R\sqrt{R_{s^R}}}{\sigma_M} \quad (3.7)$$

Where σ_M is volatility of the market return. We may now conclude that in this setting the return on defaulted debt on the s[th] exposure (or segment) r_s^D is equal to the expected return on the collateral, which is given by the risk-free rate r_{rf} and the debt-specific risk-premium δ_s :

$$\begin{aligned} r_s^D &= r_{rf}+\frac{\sigma_i^R\sqrt{R_{s^R}}}{\sigma_M}\left(r_M-r_{rf}\right) \quad (3.8) \\ &= r_{rf}+\beta_{s^R,M}MRP=r_{rf}+\delta_{s^R} \end{aligned}$$

Where the *market risk premium* is given by $MRP\equiv r_M-r_{rf}$ (also assumed to be constant through time) and the debt-specific risk premium is given by $\delta_{s^R}=\beta_{s^R,M}MRP$. This approach identifies the systematic factor with the standardized return on a market portfolio r_M , from which it follows that the asset correlation to the former can be interpreted as a normalized "beta" in a single factor CAPM (or just a correlation between the defaulted debt's and the market's return), which is given by $\rho_{i,s^R}\equiv\sqrt{R_{s^R}}$. In subsequent sections, we pursue alternative estimations $\hat{\rho}_{i,s^R}$, through regressing actual defaulted debt returns on some kind of market factor or other measure of systematic risk (e.g., aggregate default rates)[9], while controlling for firm or instrument specific covariates.

4. Empirical Methodology

We adopt a simple measure, motivated in part by the availability of a rich data-set of defaulted bonds and loans available to us, which analyzes the observable market prices of debt at two points in time: the default event (e.g., bankruptcy or other financial distress qualifying as a default) and the resolution of the default event (e.g., emergence from bankruptcy under Chapter 11 or liquidation under Chapter 7). We calculate the annualized rate of return on the i^{th} defaulted debt instrument in segment j as:

$$r_{i,j}^D=\left(\frac{P^E_{i,j,t^E_{i,j}}}{P^D_{i,j,t^D_{i,j}}}\right)^{\frac{1}{t^E_{i,j}-t^D_{i,j}}}-1 \quad (4.1)$$

where $P^D_{i,j,t^D_{i,j}}$ ($P^E_{i,j,t^E_{i,j}}$) are the prices of debt at time of default $t^D_{i,j}$ (emergence $t^E_{i,j}$). An estimate for the return the j^{th} segment (seniority class of collateral type) can then be formed as the arithmetic average across loans in that segment:

$$\overline{r}_j^D=\frac{1}{N_j^D}\sum_{i=1}^{N_j^D}\left[\left(\frac{P^E_{i,j,t^E_{i,j}}}{P^D_{i,j,t^D_{i,j}}}\right)^{\frac{1}{t^E_{i,j}-t^D_{i,j}}}-1\right] \quad (4.2)$$

Where N_j^D is the number of defaulted loans in the recovery group j. A measure of the recovery uncertainty in recovery class s is given by the sample standard error of the mean annualized return:

$$\hat{s}_{\overline{r}_j^D}=\frac{1}{N_j^D-1}\sqrt{\sum_{i=1}^{N_j^D}\left[\left\{\left(\frac{P^E_{i,j,t^E_{i,j}}}{P^D_{i,j,t^D_{i,j}}}\right)^{\frac{1}{t^E_{i,j}-t^D_{i,j}}}-1\right\}-\overline{r}_j^D\right]^2} \quad (4.3)$$

5. Empirical Results: Summary Statistics of Returns on Defaulted Debt by Segment

In this section and the following, we document our empirical results. These are based upon our analysis of defaulted bonds and loans in the Moody's Ultimate Recovery Database™ (MURD™) release as of August, 2010. This contains the market values of defaulted instruments at or near the time of default[10], as well as the values of such pre-petition instruments (or of instruments received in settlement) at the time of default resolution. This database is largely representative of the U.S. large-corporate loss experience, from the late 1980's to the present, including most of the major corporate bankruptcies occurring in this period.

Table 1 summarizes basic characteristics of simple annualized return on defaulted debt (RDD) in (4.1) by default event type (bankruptcy under Chapter 11

[9] Alternatively we can estimate the vector of parameters $\left(\mu_i,\mu_i^R,\rho_{i,s},\rho_{i,s^R},r\right)^T$ by *full-information maximum likelihood* (FIML), given a time series of default rates and realized recovery rates (see the Appendix.) The resulting estimate $\hat{\rho}_{i,s^R}$ can be used in equation (3.8) - in conjunction with estimates of the market volatility σ_M , debt-specific volatility σ_i^R , the MRP $\left(r_M-r_f\right)$ and the risk-free rate r_f - in order to derive the theoretical return on defaulted debt within this model (Machlachlan, 2004).

[10] Experts at Moody' compute an average of trading prices from 30 to 45 days following the default event, where each daily observation is the mean price polled from set of dealers with the minimum /maximum quote is thrown out.

vs. out-of-court settlement.) and instrument type (loans – broken down by term and revolving vs. bonds). Here we also show the means and standard deviations of two other key quantities: the time-to-resolution (i.e., time from default to time of resolution) and the outstanding-at-default, for both the RDD sample as well as for the entire MURD™ database (i.e., including instruments not having trading prices at default). We conclude from this that our sample is for the most part representative of the broader database. Across all instruments, average time-to-resolution is 1.6 (1.4) years and average outstanding at default is $216.4M ($151.7M) for the analysis (broader) samples.

The bottom panel of Table 1 represents the entire Moody's database, whereas the top panel summarizes the subset for which we can calculate RDD measures. The version of MURD™ that we use contains 4,050 defaulted instruments, 3,500 (or 86.4%) of which bankruptcies, and the remaining 550 are distressed restructurings. On the other hand, in the RDD sub-set, the vast majority (94.6% or 1,322) of the total (1,398) are Chapter 11. One reason for this is that the times-to-resolution of the out-of-court settlements are so short (about 2 months on average) that it is more likely that post-default trading prices at 30-45 days from default are not available. Second, many of these were extreme values of RDD, and were heavily represented in the outliers that we choose to exclude from the analysis (30 of 35 statistical outliers.)[11]

The overall average of 1,398 annualized RDDs is 28.6%, with a standard error of the mean of 3.1%, and ranging widely from -100% to 893.8%. This says that there were some very high returns – as the 95th percentile of the RDD distributions is 191%, or that in well over 70 cases investors would have more than doubled their money holding defaulted debt. We can observe this in Figure 1, the distribution of RDD, which has an extremely long tail to the right. We observe that the distribution of RDD is somewhat different in the case of out-of-court settlements as compared to bankruptcies, with respective means RDD of 37.3% for the former, and 28.1% in the latter. The standard errors of mean RDD are also much higher in the non-bankruptcy population, 15.3% for out-of-court versus 3.2% for bankruptcies. This large difference in distributional properties can be observed in the empirical distributions of RDD by default type in Figures 2.1-2.2. The data is well-represented by bank loans, 36.8% (38.1%) of the (total MURD™) sample, or 514 (1543) out of 1398 (4050) instruments. Loans appear to behave somewhat differently than bonds, having slightly higher mean and standard error of mean RDD, 32.1% and 26.4%, respectively. Figures 3.1-3.2 show the distributions of RDD by instrument type.

Table 2 summarizes the distributional properties of RDD by seniority rankings (bank loans; senior secured, unsecured and subordinated bonds; and junior subordinated bonds) and collateral types.[12] Generally, since this does not hold monotonically across collateral classes or is consistent across recovery risk measures, better secured or higher ranked instruments exhibit superior post-default return performance. However, while the standard error of mean RDD (which we can argue reflects recovery uncertainty) tends to be lower for more senior instruments, it tends to be higher for those which are better secured. Average RDD is significantly higher for secured as compared to for unsecured facilities, 34.5% vs. 23.6% respectively. Focusing upon bank loans, we see a wider split of 33.0% vs. 19.8% for secured and unsecured. However, by broad measures of seniority ranking, mean RDD exhibits a non-monotonic albeit overall increasing pattern in seniority, while standard error of RDD is decreasing in seniority. Average RDD is 32.3% and 36.6% for loans and senior secured bonds, as compared to 23.7% and 33.2% for senior secured and senior subordinated bonds, decreasing to 15.6% for junior subordinated instruments. However, while unsecured loans have lower post-default returns than secured loans, within the secured loan class we observe returns exhibit a humped pattern as collateral quality goes down in rank: an increase in RDD from 22.6% for Cash, to 46.2% for All Assets & Real Estate, to 29.0% for PP&E & Second Lien.

Table 3 summarizes RDD by two duration measures: the "time-in-distress" (TID), defined as the time (in years) from the last cash pay date to the default

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[11] Based upon extensive data analysis in the Robust Statistics package of the S-Plus statistical computing application, we determined 35 observations to be statistical outliers. The optimal cutoff was determined to be about 1,000%, above which we removed the observation from subsequent calculations. There was a clear separation in the distributions, as the minimum RDD in the outlier sub-set is about 17,000%, more than double the maximum in the non-outlier sub-set.

[12] We have 2 sets of collateral types: the 19 lowest level labels appearing in MURD™ (Guarantees, Oil and Gas Properties, Inventory and Accounts Receivable, Accounts Receivable, Cash, Inventory, Most Assets, Equipment, All Assets, Real Estate, All Non-current Assets, Capital Stock, PP&E, Second Lien, Other, Unsecured, Third Lien, Intellectual Property and Intercompany Debt), and a 6 level high level grouping of that we constructed from the (Cash, Accounts Receivables & Guarantees; Inventory, Most Assets & Equipment; All Assets & Real Estate; Non-Current Assets & Capital Stock; PP&E & Second Lien; and Unsecured & Other Illiquid Collateral.)The latter high-level groupings were developed with in consultation with recovery analysis experts at Moody's Investors Services.

state, and the "time-to-resolution" (TTR), the duration from the date of default to the resolution or settlement date. Analysis of these measures helps us to understand the term-structure of the defaulted debt returns. We examine features of RDD by quintiles of the TTR and TID distributions, where the 1st refers to the bottom fifth of durations in length, and the 5th quintile the top longest. The patterns we observe are that RDD is decreasing (albeit non-monotonically) in TTR, while it exhibits a U-shape in TID.

Table 4 summarizes RDD by the earliest available Moody's senior unsecured credit rating for the obligor. This provides some evidence returns on defaulted debt are augmented for defaulted obligors that had, at origination (or time of first public rating), better credit ratings or higher credit quality. Mean RDD generally declines as credit ratings worsen, albeit unevenly. While the average is 22.9% for the AA-A category, it goes up to 45.1% for BBB, then down to 17.9% for BB, but up again to 31.6% for B, and finally down to 21.99% for the lowest category CC-CCC

Table 5 summarizes RDD by measures of the relative debt cushion of the defaulted instrument. MULGD provides the proportion of debt either above ("degree of subordination") or below ("debt cushion") any defaulted instrument, according to the seniority rank of the class to which the instrument belongs. It has been shown that the more debt below, or the less debt above, the better is the *ultimate recovery* on the defaulted debt (Keisman et al, 2000). We can also think of this position in the capital structure in terms of "tranche safety" – the less debt above, more debt below, or the thinner the tranche, then the more likely it is that there will be some recovery. While this is not the entire story, this measure has been demonstrated to be an important determinant of ultimate recovery, so we suspect that it will have bearing on the performance of defaulted debt. Here, we offer evidence that returns on defaulted debt measured are increasing in the degree of tranche thickness or relative debt cushion, in the sense of the difference between debt below and debt above. To the end of showing this in tabular form, we define the *Tranche Safety Index* (TSI) as:

$$TSI \equiv \frac{1}{2}\left[\%\,Debt\,Below - \%\,Debt\,Above + 1\right] \quad (5.1)$$

This ranges between zero and 1, where it is near zero the greater the difference between debt above and below (i.e., the thinnest tranche or the most subordinated), and closest to unity when debt below is maximized and the debt above is nil (i.e., the thickest tranche or the greatest debt cushion). In Table 6, we examine the quintiles of the TSI, where the bottom 20th percentile of the TTI distribution represents the least protected instruments, and the top 20th percentile the most protected. Additionally, we define several dummy variables in order to capture this phenomenon, as in Brady et al. (2006). "No Debt Above and Some Debt Below" (NDA/SDB) represents a group that should be the best protected, while "Some Debt Above and Some Debt Below" (SDA/SDB) and "No Debt Above and No Debt Below" (NDA/NDB) represent intermediate groups, and "No Debt Below and Some Debt Above" (NDB/SDA) should be the least protected group. Table 6 shows that there is there is U-shape overall in average RDD with respect to quintiles of TSI: starting at 35.1% at the bottom quintile, having a minimum in the 2nd of 11.0%, and increasing thereafter to 25.8%, 42.3% and 47.5% at the top. In regard to the dummy variables, we observe a general decrease in average RDD, decreasing from the most to the least protected categories: 42.8%. 24.1%, 25.2% and 19.7% from NDA/SDB to NDA/SDA.

6. Summary Statistics and Distributional Properties of Covariates

In this section we first analyze the independent variables available to us and calculated from MURD™, as well as data attached to this from Compustat and CRSP, and then discuss a multivariate regression model to explain RDD. Table 6 summarizes the distributional properties of key covariates in our database and their univariate correlations to RDD. We have grouped these into the following categories: Financial Statement and Market Valuation, Equity Price Performance, Capital Structure, Credit Quality / Credit Market, Instrument / Contractual, Macro / Cyclical and Durations / Vintage.

The financial variables, alone or in conjunction with equity market metrics, extracted from Compustat or CRSP. The Compustat variables are taken from the date nearest to the 1st instrument default date of the obligor, but no nearer than one month, and no further than one year, to default. These are shown in the top panel of Table 8. First, we see some evidence that leverage is positively related to RDD, suggesting that firms that were nearer to their "default points" prior to the event had defaulted debt that performed better over the resolution period, all else equal. This is according to an accounting measure, Book Value of Total Liabilities / Book Value of Total Assets, which has a substantial positive correlation of 17.2%.

Next, we consider two variables measuring the degree of market valuation relative to stated value, or alternatively the degree of intangibility in assets Market Value of Total Assets / Book Value of Total Assets (MVTA/BVTA) and Book Value of Intangibles / Book Value of Total Assets. In this group, there is evidence of a positive relationship to the RDD, which is strongest by far for MVTA/BVTA, having a correlation of 18.5%. This enters into some of our candidate regression models significantly, but not the final model chosen. We speculate that the intuition here is akin to a "growth stock effect" – such types of firms may have available a greater range of investment options, that when come to fruition results in better performance of the defaulted debt on average.

We display 3 covariates in Table 6 that measure the cash-flow generating ability of the entity: Free Asset Ratio (FAR), Free Cash Flow / Book Value of Total Assets and the Cash Flow from Operations / Book Value of Total Assets. Results show generally a negative correlation between cash flow ratios and RDD, notably a strong negative correlation for FAR of 9.0%. The intuition here may be considered strained, as it is natural to think that the ability to throw off cash may signal a firm with an underlying business model that is viable, which is conducive to a successful emergence from default

and well performing debt; however, this may also be taken to mean an "excess" of cash with not good investments to apply it to and a basically poor economic position.

Finally for the financials, we have a set of variables that measure some notion of accounting profitability: Retained Earnings / Book Value of Total Assets, Return on Assets and Return on Equity. These have generally a modest inverse relation to RDD. As with other dimensions of risk considered here, we resort to a "backward story", relative to the expectation that least-bad profitability mitigates credit or default risk: that is, if already in default, then better accounting profitability may be a harbinger of deeper woes for the firm, as reflected in the performance of its debt to emergence. However, none of these enter the multiple regressions.

Equity price performance metrics, extracted from CRSP at the date nearest to the 1st default date of the obligor, but no nearer than on month to default. These are shown in the 2nd from top panel of Table 8. The 1-Month Equity Price Volatility, the standard deviation of daily equity returns in the month prior to default, exhibits a small modest positive correlation of 2.5% to RDD. This sign is explainable by an option theoretic view of recoveries, since the value of a call-option on the residual cash flows of the firms to creditors firm are expected to increase in asset value volatility, which is reflected to some degree in equity volatility. On the other hand, the 1-Year Expected Equity Return, defined as the average return on the obligor's stock in excess of the risk-free rate the year prior to default, exhibits a modest degree of negative correlation (-6.4%). We find this a little puzzling. On the other hand, the Cumulative Abnormal Returns on equity, the returns in excess of a market model in the 90 days prior to default, have the strongest positive relationship to RDD of the group, 10.3%. This is understandable, as the equity markets may have a reasonable forecast of the firm's ability to become rehabilitated in the emergence from default, as reflected in "less poor" stock price performance relative to the market. Note this is one of two variables in this group that enters the candidate regression models. Market capitalization of the firm relative to the market as a whole, defined as the logarithm of the scaled market capitalization [13], also has a significant negative univariate correlation to the market of 8.6%, and enters all of the regressions, as with CAR. We have no clear a priori expectation for this variable, perhaps we would expect larger companies to have the "resiliency" to better navigate financial distress, counter to what we are measuring. The Stock Price Relative to the Market ("SPRM"), which is the percentile ranking or the absolute level of the stock price in the market, has a moderate negative correlation to RDD of -4.4%. As the purpose of this variable is to capture the delisting effect when a stock price goes very low, we might expect the opposite sign on this correlation. Finally, the Stock Price Trading Range ("SPTR"), defined as the stock price minus its 3-year low divided by the difference between its 3-year high and 3-year low, is showing only a small negative correlation to RDD of -2.9%. This is another counter-intuitive result, as one might expect that a stock doing better as compared to its recent range to signal a better quality firm whose debt might perform better in default, but the data is not showing that, or much less of any kind of relationship here.

Capital structure metrics, extracted from the MURD™ data at the default date of the obligor, are shown in the 3rd from top panel of Table 8. The two measures of capital structure complexity, Number of Instruments ("NI") and Number of Creditor Classes ("NCC"), show an inverse relationship to defaulted debt performance. NI (NCC) has a modest negative correlation to RDD of -4.0% (-3.0%). We might expect a simpler capital structure to be conducive to favorable defaulted debt performance according to a coordination story. Note that neither of these variables enters the final regression models. While most companies in our database have relatively simple capital structures, with NI and NCC having medians of 6 and 2, respectively, there are some rather complex structures (the respective maxima are 80 and 7).

We have three variables in this group that measure the nature of debt composition: Percent Secured Debt ("PSCD"), Percent Bank Debt ("PBD") and Percent Subordinated Debt ("PSBD"). The typical firm

[13] The scale factor is defined as the market capitalization of the stock exchange where the obligor trades time 10,000.

in our database has approximately 40% to 50% of its debt either secured, subordinated or bank funded. All of these exhibit moderate positive correlation to RDD of 8.8%, 9.4% and 8.7% for PSCD, PBD and PSBD, respectively. The result on PBD may be attributed to either a monitoring, or alternatively an "optimal foreclosure boundary choice", kind of story (Carey and Gordy, 2007). However, as with the complexity variables, none of these appear in the regression model.

The credit quality / credit market metrics, extracted from the MURD™ database and Compustat near before the default date of the obligor. These are shown in the 4th from top panel of Table 8. Two of the variables in this group have, what may seem to be at first glance, counter-intuitive relationships to RDD. First, the Altman Z-Score ("AZS"), which is available in Compustat, has a relatively large negative correlation of -8.8% (note that higher values of the AZS indicate lower bankruptcy risk). Second, the LGD implied by the trading price at default – which forms the basis for the RDD calculation – exhibits a moderate *positive* correlation to RDD of 11.3%. As this variable has been shown to have predictive power for ultimate LGD (Emery et al, 2007), at first glance this relationship may seem difficult to understand. But note that the same research demonstrates that LGD at default is also an upwardly biased estimate of ultimate LGD in some sense. Therefore, we might just as well expect the opposite relationship to hold, as intuitively it may be that otherwise high quality debt may perform better on average if it is (perhaps unjustifiably) "beaten down". Indeed, LGD enters all of our regression models with this sign, and as a more influential variable than suggested by this correlation; but AZS does not make it to any of our regression models. The remaining 2 variables in this group are reflective of the Moody's ratings at the first point that the debt is rated, the Moody's Original Credit Rating Investment Grade Dummy (MOCR-IG) and, Moody's Original Credit Rating - Minor Code (MOCR-MNC; i.e., numerical codes for notched rating classes.) The only meaningful univariate result here is a positive correlation of 12.4% in the case of MOCR-IG, and this variable enters significantly into all of our candidate regression models.

Next we consider instrument / contractual metrics, extracted from the MURD™ database at the default date of the obligor. These are shown in the 3rd from bottom panel of Table 8. Consistent with the analysis of the previous section, the correlations with RDD in this group reflect the extent that instruments which are more senior, better secured or in a safer tranches experience better performance of defaulted debt. The Seniority Rank (SR) and Collateral Rank (CR) codes (which decrease numerically for higher ranks or better security) both have negative and reasonably sized correlation coefficients with RDD, -9.6% and -10.0% for SR and CR, respectively. Percent Debt Below and Percent Debt Above are positively (negatively) correlated to RDD, coefficients of 9.4% (-5.2%). And the TSI, constructed from the latter two variables as detailed in the previous section, has a significant positive correlation with RDD of 9.7%. This is consistent with our understanding that there is in fact more recovery risk associated with low expected *LGD* segments. TSI enters 2 of our 3 candidate regression models.

In this section we consider macroeconomic / cyclical metrics measured near the default date of the obligor. These are shown in the 2nd from bottom panel of Table 8. These correlations are evidence that defaulted debt returns vary procyclically, or that debt defaulting in downturn periods tends to perform better. We have measures of the aggregate default rate, extracted from Moody's Default Rate Service (DRS™) database. These are lagging 12-month default rates, with cohorts formed on an overlapping quarterly basis[14]. The four versions of this are for the all-corporate and speculative grade segments, both in aggregate and by industry. All of these have a mild, albeit significant, positive linear correlations with RDD. The Moody's All-Corporate Quarterly Default Rate ("MACQDR"), having a 6.7% correlation with RDD, is one of the systematic risk variables to enter the candidate regression models.

The next set of variables represent measures of aggregate equity and money market performance, the Fama and French (FF) portfolio returns commonly used in the finance literature, measured on a monthly basis in the month prior to instrument default[15]. These are Excess Return on the Market ("FF-ERM"), Relative Return on Small Stocks[16] ("FF-ERSS") and the Relative Return on Value Stocks[17] ("FF-ERVS"). We see that RDD is somewhat positively associated with aggregate return on the market factor FF-ERM, having a modest correlation

[14] E.g., the default rate for the 4th quarter of 2008 would represent the fraction of Moody's rated issuers in the beginning of 4Q07 that defaulted over the subsequent year. We follow the practice of adjusting for withdrawn ratings by subtracting one-half the number of withdrawn obligors from the number of available-to-default (or the denominator of the default rate.)

[15] These can be downloaded from Kenneth French's website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

[16] This is more commonly termed the *Small Minus Large* (SML) portfolio (see Fama and French, 1992.)

[17] This is more commonly termed the *High Minus Low* (HML) portfolio, meaning high vs. low market-to- book ratio (see Fama and French, 1992.)

of only 7.2%[18]. Similarly, RDD is positively but weakly related to FF-RRSS, a correlation of only 2.8%. On the other hand, RDD seems to have a small negative correlation to FF-RRVS of -4.3%. We have one more aggregate equity market variable, 2-Year Stock Market Volatility, defined as the standard deviation of the S&P 500 return in the 2-years prior to default, which shows a modest positive linear correlation to RDD of 5.7%. Note that FF-ERM is the only of these aggregate equity market variables to enter significantly in the multiple regression models. Another set of systematic variables are aggregate interest rates, the 1-Month Treasury bill yield and the 10-Year Treasury bond yield, which exhibit moderate negative correlation to RDD of -10.2% and -7.0%, respectively. However, only the 1-Month Treasury bill yield appears in the final regressions. The intuition here may be that defaulted debt performs better in low interest rate environments, which is associated with lower aggregate economic activity, as well as a higher marginal utility of consumption on the part of investors[19].

The final set of variables that we consider in this section is duration / vintage metrics, based on calculations from extracted dates in the MURD™ database. These are shown in the bottom panel of Table 8. We can conclude from this section that the duration / vintage measures that would be in one's information set at the time of instrument default are largely uninformative regarding the performance of defaulted debt. The variables that we have chosen to display include Time from Origination to Default, Time from Last Cash Pay Date to Default, Time from Default to Emergence and Time from Origination to Maturity The time to resolution has a sizable negative correlation with RDD -13.4%), but we would not use this for modelling purposes, as it is random at the time of default.

7. Multivariate Regression Analysis of Defaulted Debt Returns

In this section we discuss the construction and results of multiple regression models for RDD. In order to cope with the highly non-normal nature of the RDD distribution, we turn to the various techniques that have been employed in the finance and economics literature to classify data in models with constrained dependent variables, either qualitative or bounded in some region. However, much of the credit risk related literature has focused upon qualitative dependent variables, which the case of *probability of default* (*PD*) estimation naturally falls into. Maddala (1991, 1983) introduces, discusses and formally compares the different *Generalized Linear Models* ("GLMs"). Here we consider the case most relevant for *RDD* estimation, and that least pursued in the GLM literature. In this context, since we are dealing with a random variable in a bounded region, this is most conveniently modelled through employing a beta distribution. Therefore, we follow Mallick and Gelfand (1994), in which the *GLM link function*[20] is taken as a mixture of cumulative beta distributions, which we term the beta-link *GLM* ("BLGLM".)

The coefficient estimates and diagnostic statistics for our "leading" three models are shown in Table 7. These are determined through a combination of automated statistical procedures[21] and expert judgment, where we try to balance sometimes competing considerations of in-sample fit with the sensibility of the models. Essentially, the three models shown in Table 9 had the best fit to the sample data, while spanning what we thought was the best set of risk factors, based upon prior expectations as well as the univariate analysis. Note that there is much overlap between the models, as Model 2 differs from Model 1 by two variables (it has MV / BV instead of TL / TA, and has RSIZ), and Model 3 from Model 2 by two variables as well (FAR in lieu of TSI and *LGD*).

Across the 3 candidate models, we observe that all coefficients estimates attain a high degree of statistical significance, in almost all cases at better than the 5% level[22], and in many cases at much better than the 1% level. The number of observations for which we had all of these explanatory variables is the same for Models 1 and 2 (968), but there is a sizable drop-off for Model 3 to only 792 observations. In all cases, the likelihood functions converged to a stable global maximum.[23] Model 3 achieves the best in-sample fit by McFadden pseudo *r*-squared of 41.7%, followed by Model 2 (38.8%) and Model 1 (32.5%). In terms of maximized log-likelihood, Model 3 is far better than the others (-

18 Results for the S&P 500 return, not shown, are very similar.

19 The term spread, or the difference in a long and short term Treasury yield, was neither significant on a univariate basis or in the regressions. This held across several different choices of terms. Therefore, we do not show these results.

20 In the terminology of GLMs, the link function connects the expectation of some function of the data (usually the random variable weighted by density, in the case of the expected value) to a linear function of explanatory variables.

21 To this end, we employ an alternating direction stepwise model selection algorithm in the mass() library of R statistical software. There were 5 candidate leading models that tied as best, we eliminated 2 of them that we judged to have economically unreasonable features".

22 Moody's investment grade rating in Model 3 is on the borderline, having a p-value of 0.06, just shy of significance at the 5% level.

23 The estimation was performed in S+ 8.0 using built-in optimization routines.

504.0), and Model 1 is only slightly better than Model 2 (-592.3 vs. -594.7) in spite of having one less explanatory variable, but as these models are not nested this may not be so meaningful a comparison. Overall, we deem these to signify good fit, given the non-linearity of the problem, the relatively high dimension as well as the high level of noise in the *RDD* variable.

We now turn to the signs and individual economic significance of the variables, note that we report *partial effects* ("PEs"), which are akin to straight coefficient estimates in an ordinary least squares regression. Roughly speaking, this represents a change in the dependent variable for a unit change in a covariate, holding other variables fixed at their average sample values.[24]

First, we consider the systematic risk variables. In the case of the Moody's speculative default rate by industry, appearing in all models, we see PE's ranging in 2.05-2.25. This implies that a percentage point elevation in aggregate default rates adds about 2% in return on defaulted debt on average, all else equal, which can be considered highly significant in an economic sense. For example, the near quadrupling in default rates between 1996 and 2001 would imply an increase in expected RDD of about 12%. On the other hand, the PE's on the 1-Month Treasury yield are in the range of -0.49 to -0.37, so that debt defaulting when short-term rates are about 2% higher will experience close to 1% deterioration in performance, ceteris paribus. Second, across all three regression models, RDD has a significant (at the 5% level) and positive loading on the FF-ERM, with PE's ranging from 1.38 to 1.55, implying that a 5% increase in the aggregate equity market return augments defaulted debt returns by about 6%.

Next, we consider the contractual variables. The dummy variable for secured collateral has PE's ranging in 0.23-0.27 across models, suggesting that the presence of any kind of security can be expected to augment expected RDD by about 25%, which is an economically significant result. The TSI, appearing only in Models 1 and 2, has a PE ranging in 0.43-0.45, suggesting that going up a single decile in this measure can increase RDD by anywhere from 4% to 5%.

Turning to the credit quality / market variables, for *LGD* at default, only in Models 1 and 2, PE's are about 0.28-0.33, implying that a 10% lower expected recovery rate by the market at default can lead to about a 3% higher expected RDD. The dummy variable for a Moody's investment grade rating at origination, appearing in all models, has PE's ranging from 0.16 in Model 3 to 0.24 in Model 2. This tells us that "fallen angels" are expected to have about 15-25% better return on their defaulted debt. On the other hand, the single relative stock price performance variable CAR, in all 3 models, has PE's ranging in 0.37-0.40. This says that, for example, a firm with 10% better price performance relative to the market in the 90 days prior to default will experience about 4% better return on its defaulted debt.

In the case of the financial ratios, TL/TA appears only in Model 1, having a PE of 0.27. This means that the debt of a defaulted firm having 10% higher leverage at default will have about 3% greater return on its debt. MV/BV appears in Models 2 and 3, with respective PE's of 0.19 and 0.14, so that a 10% higher market valuation translate on average into nearly a 2% better return on defaulted debt. Finally in this group, the cash-flow measure FAR only appears in Model 3, with a PE of -0.24. This implies that if a defaulted firm has 10% greater cash generating ability by this measure, then holding other factors constant its RDD should return about 2.5% less.

Finally, the size of the firm relative to the market appears in only Models 2 and 3, with PE's of about 0.06 to 0.04. As this is in logarithmic terms, we interpret this as if a defaulted firm doubles in relative market capitalization, we should expect its RDD to be augmented by around 5%, all other factors being held constant.

In order to settle upon a "favored" or "leading" model, we performed an out-of-sample and out-of-time analysis. We re- estimated the models for different sub-samples of the available data, starting from the middle of the data-set in year 1996. We then evaluate how the model predicts the realized RDD a year ahead. We employed a resampling procedure, sampling randomly with replacement from the development data-set (e.g., the period 1987-1996), and in each iteration re-estimating the model. Then for the year ahead, we resample with replacement (e.g., the 1997 cohort), and evaluate the goodness-of-fit for the model. This is performed 1000 times, then a year is added, and this is repeated until the sample is exhausted. At the end of the procedure, we collect the *r*-squared's, and study their distribution, for each of the 3 models. The results of this show that the mean out-of-sample *r*-squared in Model 1 is highest, at 21.2%, followed by Model 3 (17.8%) and Model 2 (12.1%). On the basis of the numerical standard errors (on the order of 1-2%), we deem these to be significantly different. Given the best performance on this basis, in conjunction with other considerations, we decide that Model 1 is the best. The other reasons for choosing Model 1 are its parsimony relative to Model 2, and that it contains a credit market variable (*LGD*), the latter we believe makes for a more compelling story. Note that this procedure is robust to structural breaks, as

[24] See Maddala (1981) for a discussion of this concept in the context of probit and logit regressions.

the model is redeveloped over an economic cycle, as in each iteration the same variables are chosen, and the models display the same relative performance over time.

8. Conclusions and Directions for Future Research

In this paper, we have empirically studied the market performance of a long history of defaulted debt. We examined the distributional properties of the *return on defaulted debt* (RDD) measure across different segmentations in the dataset (e.g., default type, facility type, time period, seniority, industry), and developed a multiple regression model for RDD in the generalized linear model (GLM) class.

We found that defaulted debt returns vary significantly according to certain different factors. There is some evidence that RDD is elevated for debt having better collateral quality rank or better protected tranches within the capital structure; and for obligors rated higher at origination, larger in market capitalization relative to the market, more financially levered or having higher Cumulative Abnormal Returns on Equity (CARs) at default. However, RDD is increasing in market implied loss severity at default (loss given default - *LGD*). We also find evidence that returns vary pro-cyclically, as they are positively correlated with industry default rates, but there tends to be a lag in the relationship; further, they are inversely related to short-term interest rates, and positively related to returns on the equity market. This research is of interest to investors in distressed and defaulted debt, and may help guide in the construction of trading strategies. A natural extension and application of our model would be to quantify trading gains from implementing our model.

9. References

1. Acharya, V., Bharath, S.T and Srinivasan, A. (2007), 'Does industry-wide distress affect defaulted firms? – Evidence from creditor recoveries', Journal of Political Economy, Vol. 85, pp. 787-821.
2. Altman, E.I. (1989), 'Measuring corporate bond mortality and performance', Journal of Finance, Vol. 44, pp. 909-922.
3. Altman, E.I., Resti, A. and Sironi, A. (2003) Default Recovery Rates in Credit Risk Modelling: A Review of the Literature and Empirical Evidence, Working Paper, NYU Solomon Center.
4. Altman, E.I. and Jha, S. (2003) Market Size and Investment Performance of Defaulted Bonds and Loans: 1987-2002, Working Paper, NYU Stern School of Business.
5. Altman, E.I. (2010) Defaulted Debt Indexes Newsletter, NYU Stern School of Business (February).
6. The Basel Committee on Banking Supervision (BCBS) (2004) International Convergence on Capital Measurement and Capital Standards, Basel, Switzerland (June).
7. Altman, E.I. (2005) Guidance on paragraph 468 of the framework document, Basel. Switzerland (July.)
8. Barco, M. (2007), 'Going downturn', Risk, September, pp. 38-44.
9. Carey, M. and Gordy, M. (2007) The Bank as Grim Reaper: Debt Decomposition and Recoveries on Defaulted Debt, Working Paper, The Board of Governors of the Federal Reserve System.
10. Davydenko, S. and Strebuleav, I. (2002) Strategic Behavior, Capital Structure and Credit. Spreads: An Empirical Investigation, Working Paper, London Business School.
11. Dullman, K. and Trapp, M. (2004) Systematic Risk in *LGD*: An Empirical Analysis, Working Paper, University of Mannheim.
12. Emery, K., Cantor, R., Keisman, D. and Ou, S. (2007) Moody's Ultimate Recovery Database: Special Comment, Moody's Investor Service (April).
13. Fama, E.F. and French, K.R. (1992), 'The cross-section of expected stock returns', Journal of Finance, Vol. 47, pp. 42-480.
14. Frye, J. (2000a), 'Collateral damage', Risk, April, pp. 91-94.
15. Frye, J. (2000b) Collateral Damage Detected, Federal Reserve Bank of Chicago, Emerging Issues Series, October, pp. 1-14.
16. Frye, J. (2000c), 'Depressing recoveries', Risk, November, pp. 108-111.
17. Frye, J. (2003), 'A false sense of security'. Risk, August, pp. 63-67.
18. Giese, G. (2005), 'The impact of *PD*/*LGD* correlations on credit risk capital', Risk, April, pp. 79-84.
19. Gilson, S. (1955), 'Investing in distressed situations: a market survey', Financial Analysts Journal, November-December, pp. 8-27.
20. Gordy, M. (2003), 'A risk-factor model foundation for ratings-based bank capital rules', Journal of Financial Intermediation, Vol. 12, pp. 199-232.
21. Guha, R. (2003), 'Recovery of face value at default: empirical evidence and implications for

credit risk pricing', Working Paper, London Business School.
22. Hillebrand, M. (2006), 'Modelling and estimating dependent loss given default, Risk, September, pp. 120-125.
23. Hotchkiss, E.S. and Mooradian, R.M. (1997), 'Vulture investors and the market for control of distressed firms', Journal of Financial Economics, Vol. 43, pp. 401-432.
24. Hu, Y.T. and Perraudin, W. (2002) The Dependence of Recovery Rates and Defaults, Working Paper, Bank of England.
25. Keenan S.C., Hamilton, D.T. and Berthault, A. (2000) Historical Default Rates of Corporate Bond Issuers: 1920-1999, Moody's Investors Services (January.)
26. Keisman, D. and van de Castle, K. (2000) Suddenly Structure Mattered: Insights into Recoveries of Defaulted Debt (Corporate Ratings – Commentary), Standard and Poors.
27. Maclachlan, I. (2004) Choosing the Discount Factor for Estimating Economic *LGD*, Working Paper, Australia and New Zealand Banking Group Ltd.
28. Maddala, G.S. (1983) Limited dependent and qualitative variables in finance, 3rd ed., Cambridge, MA: Cambridge University Press.
29. Maddala, G.S. (1991), 'The perspective on the use of limited-dependent and qualitative variables models in accounting research', The Accounting Review, Vol. 66, pp. 788-807.
30. Mallick, B.K. and Gelfand, A.E. (1994), 'Generalized linear models with unknown link functions', Biometrika, Vol. 81, pp. 237-245.
31. Merton, R.C. (1971), 'Optimum consumption and portfolio rules in a continuous-time model', Journal of Economic Theory, Vol. 3, pp. 373-413.
32. Merton, R.C. (1974), 'On the pricing of corporate debt: The risk structure of interest rates', Journal of Finance, Vol. 29, pp. 449-470.
33. Pykhtin, M. (2003), 'Unexpected recovery risk', Risk, August, pp. 74-79.
34. Rosch, D. and Scheule, H. (2005), 'A multifactor approach for systematic default and recovery risk', Risk, September, pp. 62-75.
35. Schuermann, T. (2003) What do we know about loss given default?, in D. Shimko, ed.: Credit Risk Models and Management, 2nd Edition, London, UK: Risk Books.
36. Vasicek, O. (1987) Probability of Loss on a Loan Portfolio, Working Paper, KMV Corporation.
37. Vasicek, O. (2002), 'Loan portfolio value', Risk, December, pp.160-162.

10. Appendix

In this appendix we outline the development of the likelihood function for a 2-factor extension of the asymptotic single risk factor model (Gordy, 2003), which underlies the Basel 2 framework for regulatory credit risk capital, incorporating systematic recovery risk. Let us denote asset value for the r^{th} segment of firms (these can be rating classes) by:

$$A_{t,r} = \rho_r X_t + \sqrt{1-\rho_r^2} Z_{t,r} \qquad (10.1)$$

where $A_{t,r}$ is the asset value of the representative borrower in rating class r at time t, $Z_{t,r} \sim N(0,1)$ is the corresponding idiosyncratic risk factor, $X_t \sim N(0,1)$ (and independent of $Z_{t,r}$) is the systematic risk factor governing aggregate default rates at time t and the non-negative parameter ρ_r is the sensitivity (or loading) of assets in class r to the systematic risk factor (and ρ_r^2 is referred to as the asset value correlation). It follows that the conditional probability-of-default (*PD*) is given by the ubiquitous Vasicek (1987) formula:

$$\begin{aligned} R(X_t \mid PD_r, \rho_r) &= \Pr\left(Z_{t,r} < \frac{\Theta^{-1}(PD_r) - \rho_r X_t}{\sqrt{1-\rho_r^2}} \right) \\ &= \Theta\left(\frac{\Theta^{-1}(PD_r) - \rho_r X_t}{\sqrt{1-\rho_r^2}} \right) \end{aligned} \qquad (10.2)$$

Where $R(X_t \mid PD_r, \rho_r)$ denotes the conditional *PD* as a function of the systematic risk factor, where PD_r is the unconditional (or long-run) probability-of-default parameter for the r^{th} rating, $\Phi(z) = \Pr(Z \le z) = \frac{1}{2\pi}\int_{-\infty}^{z} e^{-\frac{1}{2}u^2} du$ is the cumulative distribution function for a standard normal random variable and $\Phi^{-1}(p) = \inf_z \Pr(p \le \Phi(z))$ is the inverse of the distribution (or the quantile function). We can derive the distribution of the default rate (the realization of the conditional *PD*) in year t for rating class r, $dr_{t,r} \equiv L(x_t \mid PD_r, \rho_r)$, by a change-of-variables technique, as (10.2) is invertible. The systematic risk factor is:

$$X_t = \frac{\Phi^{-1}(PD_r) - \sqrt{1-\rho_r^2}\,\Phi^{-1}(dr_{t,r})}{\rho_r} \qquad (10.3)$$

Then, according to the formula

$$X \sim f_X(x), y = g(x) \rightarrow Y \sim f_Y(y) = \left|\frac{dg^{-1}(y)}{dy}\right| f_X\left(g^{-1}(y)\right), \quad (10.4)$$

the distribution of $dr_{t,r}$:

$$f_{dr}\left(dr_{t,r} \mid PD_r, \rho_r\right) = \frac{\sqrt{1-\rho_r^2}}{\rho_r \phi\left(dr_{t,r}\right)} \phi\left(\frac{\Phi^{-1}(PD_r) - \sqrt{1-\rho_r^2}\Phi^{-1}\left(dr_{t,r}\right)}{\rho_r}\right). \quad (10.5)$$

Where $\varphi(z) = \frac{1}{2\pi} e^{-\frac{1}{2}z^2}$ is the normal density function. We model the recovery side analogously, starting with a "loss process" $L_{t,s}$ at time t for seniority class s:

$$L_{t,s} = \rho_s Y_t + \sqrt{1-\rho_s^2} Z_{t,s} \quad (10.6)$$

where $Z_{t,s} \sim N(0,1)$ is the corresponding idiosyncratic risk factor, $Y_t \sim N(0,1)$ (and independent of $Z_{t,s}$) is the systematic risk factor governing loss rates at time *t* and the non-negative parameter ρ_s is the sensitivity (or loading) of assets in seniority class s to the systematic risk factor (and ρ_s^2 is the "loss correlation"). It follows that the conditional loss-given-default (*LGD*) is given by the ubiquitous Vasicek (1987) formula:

$$L(Y_t \mid LGD_s, \rho_s) = \Pr\left(Z_{t,s} < \frac{\Theta^{-1}(LGD_s) - \rho_s Y_t}{\sqrt{1-\rho_s^2}}\right) = \Theta\left(\frac{\Theta^{-1}(LGD_s) - \rho_s Y_t}{\sqrt{1-\rho_s^2}}\right) \quad (10.7)$$

where $L(y_t \mid LGD_s, \rho_s)$ denotes the *LGD* conditional as a function of the systematic risk factor Y, LGD_c is the unconditional (or long-run) loss-given-default parameter for the s^{th} seniority class. We can derive the distribution of the loss rate (the realization of the conditional LGD) in year t for seniority class *s*), $lr_{t,s} \equiv L(x_t \mid PD_r, \rho_r)$, by a change-of-variables technique, as (10.7) is invertible. The systematic risk factor is:

$$Y_t = \frac{\Phi^{-1}(LGD_s) - \sqrt{1-\rho_s^2}\Phi^{-1}\left(lr_{t,s}\right)}{\rho_s} \quad (10.8)$$

Then, according to the (10.4) distribution of $lr_{t,s}$:

$$f_{lr}\left(lr_{t,s} \mid LGD_s, \rho_s\right) = \frac{\sqrt{1-\rho_s^2}}{\rho_s \phi\left(lr_{t,s}\right)} \phi\left(\frac{\Phi^{-1}(LGD_s) - \sqrt{1-\rho_s^2}\Phi^{-1}\left(lr_{t,s}\right)}{\rho_s}\right) \quad (10.9)$$

We now derive the likelihood function for the model parameters. We assume that the systematic risk factors on the *PD* and *LGD* sides, X_t and Y_t, are standardized bivariate normal with correlation r_{XY}:

$$\left(X_t, Y_t\right)^T \sim \Phi_2\left(\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}1 & r_{XY}\\ r_{XY} & 1\end{pmatrix}\right) \quad (10.10)$$

where the $\Phi_2(x, y \mid \rho) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(x^2 - 2\rho xy + y^2\right)$ is bivariate standardized normal distribution for zero-mean and unit variance random variables *x*, *y* with correlation ρ. The likelihood contribution for a representative instrument in r^{th} rating and s^{th} seniority class in year t has the form:

$$l\left(dr_{t,r}, dr_{t,s} \mid PD_r, \rho_r, LGD_s, \rho_s, r_{XY}\right) = \frac{1}{2\pi\sqrt{1-r_{XY}^2}} \times \exp\left(\begin{array}{l} \left(\frac{\Phi^{-1}(PD_r) - \sqrt{1-\rho_r^2}\Phi^{-1}\left(dr_{t,r}\right)}{\rho_r}\right)^2 - \\ 2r_{XY}\left(\frac{\Phi^{-1}(PD_r) - \sqrt{1-\rho_r^2}\Phi^{-1}\left(dr_{t,r}\right)}{\rho_r}\right) \times \left(\frac{\Phi^{-1}(LGD_s) - \sqrt{1-\rho_s^2}\Phi^{-1}\left(lr_{t,s}\right)}{\rho_s}\right) + \\ \times\left(\frac{\Phi^{-1}(LGD_s) - \sqrt{1-\rho_s^2}\Phi^{-1}\left(lr_{t,s}\right)}{\rho_s}\right)^2 \end{array}\right) \quad (11.11)$$

Assuming independence across ratings, seniorities and years, the full log-likelihood is given by:

$$LogL\left(\left\{dr_{t,r}, dr_{t,s}\right\}_{t=,..,T;r=1,..,R;s=1,..,S} \mid \left\{PD_r, \rho_r, LGD_s, \rho_s\right\}_{t=,..,T;r=1,..,R;s=1,..,S}, r_{XY}\right) = \\ = \sum_{t=1}^{T}\sum_{r=1}^{R}\sum_{s=1}^{S} \log\left[l\left(dr_{t,r}, dr_{t,s} \mid PD_r, \rho_r, LGD_s, \rho_s, r_{XY}\right)\right] \tag{10.12}$$

Table 1: Characteristics of Return on Defaulted Debt (RDD)[1] Observations by Default and Instrument Type (Moody's Ultimate Recovery Database 1987-2010)

			Bankruptcy					Out-of-Court					Total				
			Cnt	Average	Std Err of the Mean	Minimum	Maximum	Cnt	Average	Std Err of the Mean	Minimum	Maximum	Cnt	Average	Std Err of the Mean	Minimum	Maximum
Sub-population of Moody's Recoveries Database Having Trading Price of Debt at Default	Bonds and Term Loans	RDD		28,32%	3,47%	-100,00%	893,76%		45,11%	19,57%	-91,87%	846,73%		29,19%	3,44%	-100,00%	893,76%
		Time-to-Resolution[2]	1072	1,7263	0,0433	0,0027	9,0548	59	6,65%	3,33%	0,27%	144,38%	1131	1,6398	0,0425	0,0000	9,0548
		Principal at Default[3]		207 581	9 043	163	4 600 000		416 751	65 675	6 330	2 250 000		218 493	9 323	0	4 600 000
	Bonds	RDD		25,44%	3,75%	-100,00%	893,76%		44,22%	21,90%	-91,87%	846,73%		26,44%	3,74%	-100,00%	893,76%
		Time-to-Resolution[2]	837	1,4089	0,0436	0,0548	9,0548	47	0,2044	0,0786	0,0027	1,4438	884	1,3194	0,0427	0,0027	9,0548
		Principal at Default[3]		205 028	10 590	0	4 000 000		432 061	72 727	6 330	2 250 000		207 647	10 325	0	4 000 000
	Revolvers	RDD		26,93%	7,74%	-100,00%	893,76%		10,32%	4,61%	-0,04%	61,18%		25,88%	7,26%	-100,00%	893,76%
		Time-to-Resolution[2]	250	1,4089	0,0798	0,0548	9,0548	17	0,0027	0,0000	0,0027	0,0027	267	1,3194	0,0776	0,0027	9,0548
		Principal at Default[3]		205 028	19 378	0	4 000 000		246 163	78 208	32 000	1 250 000		207 647	18 786	0	4 000 000
	Loans	RDD		32,57%	5,71%	-100,00%	893,76%		26,161%	18,872%	-91,87%	532,76%		32,21%	5,49%	-100,00%	893,76%
		Time-to-Resolution[2]	485	1,4089	0,0548	0,0027	9,0548	29	18,12%	9,96%	0,0027	2,8959	514	1,2458	0,0743	0,0027	9,0548
		Principal at Default[3]		193 647	11 336	0	4 000 000		291 939	78 628	24 853	1 750 000		199 192	16 088	0	4 000 000
	Total	RDD		28,05%	3,17%	-100,00%	893,76%		37,33%	15,29%	-91,87%	846,73%		28,56%	3,11%	-100,00%	893,76%
		Time-to-Resolution[2]		1,6663	0,0384	0,0027	9,0548		0,0522	0,0260	0,0000	1,4438		1,5786	0,0376	0,0000	9,0548
			1322					76					1398				
		Principal at Default[3]		207 099	8 194	0	4 600 000		378 593	54 302	0	2 250 000		216 422	8 351	0	4 600 000

Entie Population of Moody's Recoveries Database	Bonds and Term Loans	Time-to-Resolution[2]	2798	1,6982	0,0253	0,0027	9,3151	433	0,2084	0,0261	0,0027	3,8767	3231	1,4986	0,0239	0,0027	9,3151
		Principal at Default[3]		149 623	4 585	0	4 600 000		204 750	16 469	0	3 000 000		157 011	4 553	0	4 600 000
		Discounted LGD[3]		48,57%	0,83%	-69,78%	100,00%		14,50%	1,37%	-27,66%	100,00%		43,83%	0,78%	-69,78%	100,00%
	Bonds	Time-to-Resolution[2]	2162	1,7786	0,0290	0,0027	9,3151	345	0,2084	0,0292	0,0027	3,8767	2507	1,5620	0,0275	0,0027	9,3151
		Principal at Default[3]		157 488	5 608	0	4 600 000		204 750	18 450	0	3 000 000		166 781	5 551	0	4 600 000
		Discounted LGD[3]		39,47%	1,47%	-69,78%	100,00%		18,00%	2,76%	-3,58%	100,00%		36,40%	1,35%	-69,78%	100,00%
	Revolvers	Time-to-Resolution[4]	702	1,3944	0,1062	0,0027	9,0548	117	0,1490	0,0476	0,0027	2,8959	819	1,2165	0,0407	0,0027	9,0548
		Principal at Default[5]		131 843	21 396	0	4 000 000		124 199	17 836	347	1 250 000		130 751	226 241	0	4 000 000
		Discounted LGD[3]		40,03%	1,08%	-69,78%	100,00%		17,20%	2,03%	-27,66%	100,00%		37,00%	0,99%	-69,78%	100,00%
	Loans	Time-to-Resolution[4]	1338	1,4089	0,0330	0,0027	9,0548	205	0,1812	0,0375	0,0027	2,8959	1543	1,2458	0,0309	0,0027	9,0548
		Principal at Default[5]		127 586	5 521	0	4 000 000		124 671	14 739	347	1 750 000		127 199	5 171	0	4 000 000
		Discounted LGD[3]		45,31%	0,66%	-69,78%	100,00%		15,25%	1,13%	-27,66%	100,00%		41,23%	0,61%	-69,78%	100,00%
	Total	Time-to-Resolution[4]	3500	1,6373	0,0221	0,0027	9,3151	550	0,1958	0,0026	0,0027	3,8767	4050	1,4415	0,0208	0,0027	9,3151
		Principal at Default[5]		146 057	4 064	0	4 600 000		187 615	13 576	0	3 000 000		151 701	3 972	0	4 600 000

Notes:

1 - "Return on Defualted Debt»: annnualized simple rate of return on defaulted debt from just after the time of default (1st trading date of debt) until the time of ultimate resolution.
2 - The total instrument outstanding at default.
3 - The time in years from the instrument default date to the time of ultimate recovery.

Table 2: Return on Defaulted Debt (RDD[1]) by Seniority Ranks and Collateral Types (Moody's Ultimate Recovery Database 1987-2010)

		Revolving Credit / Term Loan			Senior Secured Bonds			Senior Unsecured Bonds			Senior Subordinated Bonds			Subordinated Bonds			Total Instrument		
	Collateral Type	Cnt	Avg	Std Err	Cnt	Avg	Std Err	Cnt	Avg	Std Err	Cnt	Avg	Std Err	Cnt	Avg	Std Err	Cnt	Avg	Std Err
Minor Collateral Category	Guarantees	2	-96,0%	4,0%	0	N/A	N/A	0	N/A	N/A	0	N/A	N/A	0	N/A	N/A	2	-96,0%	4,0%
	Oil and Gas Properties	2	77,5%	68,3%	0	N/A	N/A	0	N/A	N/A	0	N/A	N/A	0	N/A	N/A	2	77,5%	68,3%
	Inventory and Accounts Receivable	28	20,2%	23,4%	0	N/A	N/A	0	N/A	N/A	0	N/A	N/A	0	N/A	N/A	28	20,2%	23,4%
	Accounts Receivable	5	24,5%	28,5%	2	23,9%	40,6%	0	N/A	N/A	0	N/A	N/A	0	N/A	N/A	7	24,4%	21,6%
	Cash	2	114,8%	17,0%	0	N/A	N/A	0	N/A	N/A	0	N/A	N/A	0	N/A	N/A	2	114,8%	17,0%
	Inventory	1	-100,0%	N/A	1	29,3%	N/A	0	N/A	N/A	0	N/A	N/A	0	N/A	N/A	2	-35,3%	64,7%
	Most Assets	6	25,6%	30,4%	1	161,5%	N/A	0	N/A	N/A	0	N/A	N/A	1	72,1%	N/A	8	48,4%	28,1%
	Equipment	1	-100,0%	N/A	17	41,9%	8,9%	0	N/A	N/A	0	N/A	N/A	0	N/A	N/A	18	34,0%	11,5%
	All Assets	363	32,4%	6,9%	36	33,4%	23,8%	1	86,5%	N/A	0	N/A	N/A	0	N/A	N/A	400	32,6%	6,6%
	Real Estate	4	132,0%	73,6%	2	63,8%	110,9%	0	N/A	N/A	1	57,4%	N/A	0	N/A	N/A	7	101,8%	48,3%
	All Non-current Assets	2	-41,7%	58,3%	3	-60,7%	35,1%	0	N/A	N/A	0	N/A	N/A	0	N/A	N/A	5	-53,1%	27,0%
	Capital Stock	36	40,0%	15,4%	38	65,2%	19,1%	0	N/A	N/A	0	N/A	N/A	0	N/A	N/A	74	52,9%	12,3%
	PP&E	8	106,0%	70,7%	17	6,9%	17,4%	0	N/A	N/A	0	N/A	N/A	0	N/A	N/A	25	38,6%	26,3%
	Second Lien	21	23,2%	26,8%	17	24,1%	18,4%	1	119,6%	N/A	1	-46,0%	N/A	0	N/A	N/A	40	24,3%	16,2%
	Other	0	N/A	N/A	1	-24,7%	N/A	0	N/A	N/A	0	N/A	N/A	0	N/A	N/A	1	-24,7%	N/A
	Unsecured	32	19,8%	7,9%	3	-27,7%	36,6%	452	23,7%	4,9%	158	31,0%	10,2%	117	15,7%	11,1%	762	23,6%	4,0%
	Third Lien	1	106,1%	N/A	1	4,9%	N/A	7	3,5%	22,3%	1	439,4%	N/A	2	-21,8%	2,5%	12	44,3%	39,2%
	Intellectual Property	0	N/A	N/A	2	28,6%	43,9%	0	N/A	N/A	0	N/A	N/A	0	N/A	N/A	2	28,6%	43,9%
	Intercompany Debt	0	N/A	N/A	1	143,1%	N/A	0	N/A	N/A	0	N/A	N/A	0	N/A	N/A	1	143,1%	N/A
Major Collateral Category	Cash, Accounts Receivables & Guarantees	39	22,6%	18,2%	2	23,9%	40,6%	0	N/A	N/A	0	N/A	N/A	0	N/A	N/A	41	22,6%	17,4%
	Inventory, Most Assets & Equipment	8	-5,8%	30,3%	19	47,5%	10,2%	0	N/A	N/A	0	N/A	N/A	1	72,1%	N/A	28	33,2%	11,7%
	All Assets & Real Estate	367	33,5%	6,9%	38	35,0%	22,9%	1	86,5%	N/A	1	57,4%	N/A	0	N/A	N/A	407	33,8%	6,6%
	Non-Current Assets & Capital Stock	38	35,7%	15,0%	41	56,0%	18,6%	0	N/A	N/A	0	N/A	N/A	0	N/A	N/A	79	46,2%	12,0%
	PPE & Second Lien	29	46,1%	27,6%	35	14,3%	12,3%	1	119,6%	N/A	1	-46,0%	N/A	0	N/A	N/A	66	29,0%	13,9%
	Unsecured & Other Illiquid Collateral	33	22,4%	8,1%	7	17,4%	28,5%	459	23,4%	4,8%	159	33,6%	10,4%	119	15,1%	10,9%	777	24,1%	3,9%

Total Unsecured	32	19,8%	7,9%	3	27,7%	36,6%	452	23,7%	4,9%	158	31,0%	10,2%	117	15,7%	11,1%	762	23,6%	4,0%
Total Secured	482	33,0%	5,8%	139	38,0%	9,1%	9	25,6%	22,6%	3	150,3%	147,6%	3	9,5%	31,4%	636	34,5%	4,9%
Total Collateral	514	32,2%	5,5%	161	36,6%	8,4%	461	23,7%	4,8%	161	33,2%	10,3%	120	15,6%	10,8%	1398	28,6%	3,1%

Note:
1 - Annualized "Return on Defaulted Debt" from the time of default until the time of ultimate resolution.

Table 3: Returns on Defaulted Debt (RDD) [1] of Defaulted Instruments by Quintiles of Time-to-Resolution (TTR)[2] and Time-in-Distress (TID)[3] from Last Cash Pay to Default Date (Moody's Ultimate Recovery Database 1987-2010)

		Quintiles of Time from Default to Resolution Date											
		1		2		3		4		5		Total	
		Average	Std Err of the Mean	Average	Std Err of the Mean	Average	Std Err of the Mean	Average	Std Err of the Mean	Average	Std Err of the Mean	Average	Std Err of the Mean
Quintiles of Time from Last Cash Pay to Default Date	1	64,19%	24,57%	25,75%	26,11%	38,32%	13,54%	29,75%	14,08%	-4,99%	9,21%	35,04%	9,66%
	2	22,10%	15,41%	38,34%	17,09%	28,24%	19,03%	26,69%	9,21%	8,23%	6,82%	25,93%	6,78%
	3	20,81%	12,16%	30,55%	18,16%	10,04%	8,12%	27,19%	11,21%	8,90%	5,06%	19,28%	5,26%
	4	91,53%	31,75%	41,38%	19,92%	19,79%	9,16%	23,55%	6,26%	8,96%	3,91%	28,67%	5,51%
	5	92,08%	34,68%	57,99%	20,85%	8,82%	8,21%	34,22%	20,16%	-2,97%	8,57%	38,32%	9,23%
	Total	58,90%	11,57%	39,71%	8,89%	20,02%	5,71%	27,32%	4,99%	6,03%	2,61%	28,56%	3,11%

Notes:
1 - Annualized "Return on Defaulted Debt" from just after the time of default (1st trading date of debt) until the time of ultimate resolution.
2 - TTR: Duration in years from the date of default (bankruptcy filing or other default) to the date of resolution (emergence from bankruptcy or other settllement).
3 - TID: Duration in years from the date of the last interest payment to the date of default (bankruptcy filing or other default).

Table 4: Returns on Defaulted Debt[1] of Defaulted Instruments by Credit Rating at Origination (Moody's Ultimate Recovery Database 1987-2010)

		Count	Average of RDD	Standard Error of Mean RDD
Rating Groups	AA-A	146	22,94%	5,04%
	BBB	586	45,09%	13,25%
	BB	285	17,92%	5,24%
	B	65	31,57%	5,66%
	CC-CCC	125	21,99%	8,29%
	Investment Grade (BBB-A)	211	29,77%	8,43%
	Junk Grade (CC-BB)	996	23,71%	5,94%
	Total	1398	28,56%	3,34%

Note:
1 - Annnualized "Return on Defaulted Debt" (RDD) from just after the time of default (1st trading date of debt) until the time of ultimate resolution.

Table 5: Returns on Defaulted Debt[1] of Defaulted Instruments by Tranche Safety Index[2] (TSI) Quintiles and Categories (Moody's Ultimate Recovery Database 1987-2010)

		Count	Average of RDD	Standard Error of Mean RDD
Debt Tranche Groups	1st Quintile TSI	172	35,06%	12,88%
	2nd Quintile TSI	373	10,98%	4,85%
	3rd Quintile TSI	413	25,77%	5,40%
	4th Quintile TSI	342	42,41%	6,33%
	5th Quintile TSI	98	47,48%	9,57%
	NDA / SDB[3]	449	42,77%	4,89%
	SDA / SDB[4]	259	24,06%	7,65%
	NDA / NDB[5]	164	25,23%	9,44%
	NDB / SDA[6]	526	19,67%	5,25%
	Total	1398	28,56%	3,11%

Note:
1 - Annnualized "Return on Defaulted Debt" (RDD) from just after the time of default (1st trading date of debt) until the time of ultimate resolution.

2 - An index of the tranche safety calculated as TTS=(% Debt Below-% Debt Above+1) / 2.

3 - No Debt Above & Some Debt Below.

4 - Some Debt Above & Some Debt Below.

5 - No Debt Above & No Debt Below.

6 - No Debt Below & Some Debt Above.

Table 6: Summary Statistics on Selected Variables and Correlations with RDD[1] (Moody's Ultimate Recovery Database 1987-2010)

Category	Variable	Count	Minimum	Median	Mean	Maximum	Std Err of the Mean	Correlation with RDD	P-Value of Correlation
Financial Statement and Market Valuation	Book Value Total Liabilities / Book Value Total Assets	1106	38,00%	115,00%	137,42%	392,00%	2,33%	17,20%	4,32E-04
	Market-to-Book (Market Value Assets / Book Value Assets)	1106	44,00%	123,00%	152,61%	673,00%	2,71%	18,50%	6,34E-05
	Intangibes Ratio (Book Value Intangibles / Book Value Assets)	773	0,00%	18,34%	21,02%	87,85%	0,75%	11,91%	4,20E-04
	Free Asset Ratio	941	-95,51%	9,24%	5,89%	95,86%	1,18%	-8,97%	2,34E-03
	Free Cash Flow / Book Value of Total Assets	1006	-107,64%	-1,41%	-10,77%	34,61%	0,70%	-2,42%	6,11E-04
	Cash Flow from Operations / Book Value of Total Assets	1014	(669,12)	(0,48)	57,09	7 778,00	30,65	-3,32%	8,33E-04
	Retained Earnings / Book Value of Total Assets	1031	-757,97%	-25,80%	-61,21%	56,32%	3,01%	-5,91%	1,47E-03
	Return on Assets	1031	-159,12%	-8,52%	-22,18%	36,35%	0,92%	-6,50%	1,62E-03
	Return on Equity	1031	-2950,79%	3,10%	23,11%	6492,67%	17,19%	-4,31%	1,07E-03
Equity Price Performance	1-Year Expected Return on Equity	1106	-132,00%	-80,00%	-72,40%	161,00%	1,26%	-6,42%	1,07E-03
	1-Month Equity Price Volatility	1106	13,00%	209,00%	259,49%	6116,00%	11,18%	2,48%	1,14E-04
	Relative Size (Market Cap of Firm to the Market)	1106	-17,3400	-12,7200	-13,0487	-6,9300	0,0599	8,60%	1,31E-03
	Relative Stock Price (Percentile Ranking to Market)	1106	0,47%	11,00%	13,76%	81,00%	0,42%	-4,36%	1,05E-03
	Stock Price Trading Range (Ratio of Current to 3 Yr High/Low)	1106	0,00%	0,71%	2,95%	88,00%	0,22%	-2,92%	7,02E-04
	Cumulative Abnormal Returns (90 Days to Default)	1171	-127,70%	0,00%	-4,87%	147,14%	0,84%	10,30%	2,42E-03
Capital Structure	Number of Instruments	4050	0,0000	6,0000	9,9511	80,0000	0,1938	-4,04%	5,07E-04
	Number of Creditor Classes	4050	0,0000	2,0000	2,4669	7,0000	0,0188	-2,98%	3,74E-04
	Percent Secured Debt	4050	0,00%	47,79%	47,13%	100,00%	0,56%	8,76%	1,10E-03
	Percent Bank Debt	4050	0,00%	44,53%	45,23%	100,00%	0,54%	9,44%	1,19E-03
	Percent Subordinated Debt	4050	0,00%	41,67%	43,26%	100,00%	0,53%	8,68%	1,09E-03
Credit Quality / Credit Market	Altman Z-Score	793	-8,5422	0,3625	-0,3258	4,6276	0,0804	-8,75%	2,49E-03
	LGD at Default	1433	-8,50%	59,00%	55,05%	99,87%	0,83%	-11,28%	2,38E-04
	Moody's Original Credit Rating Investment Grade Dummy	3297	0,0000	0,0000	0,2014	1,0000	0,0070	12,40%	2,37E-04
	Moody's Original Credit Rating (Minor Code)	3342	3,0000	14,0000	12,4054	20,0000	0,0588	3,63%	5,01E-04

Instrument / Contractual	Seniority Rank	4050	1,0000	1,5000	1,7262	7,0000	0,0142	-9,60%	2,28E-04
	Collateral Rank	4050	1,0000	6,0000	4,5879	6,0000	0,0254	-10,00%	5,29E-04
	Percent Debt Below	4050	0,00%	9,92%	25,89%	100,00%	0,48%	9,36%	1,18E-03
	Percent Debt Above	4050	0,00%	0,00%	21,41%	100,00%	0,45%	-5,16%	6,48E-04
	Tranche Safety Index	4050	0,00%	50,00%	52,24%	100,00%	0,40%	9,70%	1,04E-03
Macro / Cyclical	Moody's All-Corporate Quarterly Default Rate	1322	0,00%	7,05%	7,14%	13,26%	0,09%	6,68%	1,47E-03
	Moody's Speculative Quarterly Default Rate	1322	1,31%	7,05%	7,16%	13,26%	0,09%	6,40%	1,41E-03
	Fama-French Excess Return on Market Factor	4050	-1076,00%	77,00%	31,06%	1030,00%	7,20%	7,22%	3,02E-04
	Fama-French Relative Return on Small Stocks Factor	4050	-2218,00%	31,00%	20,15%	843,00%	6,00%	2,81%	3,52E-04
	Fama-French Excess Return on Value Stock Factor	4050	-912,00%	54,00%	79,35%	1380,00%	5,75%	-4,27%	5,35E-04
	Short-Term Interest Rates (1-Month Treasury Yields)	1322	6,00%	32,00%	31,75%	79,00%	0,46%	-10,22%	2,26E-03
	Long-Term Interest Rates (10-Month Treasury Yields)	1106	332,00%	535,00%	538,42%	904,00%	3,61%	-7,00%	1,69E-03
	Stock-Market Volatility (2-Year IDX)	1106	4,00%	9,00%	10,03%	19,00%	0,12%	5,70%	1,37E-03
Durations / Vintage	Time from Origination to Default	3521	0,2500	2,9096	4,0286	29,9534	0,0631	0,57%	7,68E-05
	Time from Last Cash-Pay Date to Default	4050	0,0000	0,2384	0,3840	4,3808	0,0075	4,49%	5,63E-04
	Time from Default to Resolution	4050	0,0027	1,1534	1,4415	9,3151	0,0208	-13,41%	1,70E-03
	Time from Origination to Maturity Date	3521	0,1000	7,5890	8,9335	50,0329	0,1111	-0,85%	1,14E-04

Note:
1 - Annnualized "Return on Defaulted Debt" (RDD) from just after the time of default (1st trading date of debt) until the time of ultimate resolution.

Table 7: Beta-Link Generalized Linear Model for Annualized Returns on Defaulted Debt[1] (Moody's Ultimate Recovery Database 1987-2008)

	Model 1		Model 2		Model 3	
Variables	Partial Effect	P-Value	Partial Effect	P-Value	Partial Effect	P-Value
Intercept	0,3094	1,42E-03	0,5101	9,35E-04	0,4342	6,87E-03
Moody's 12 Month Lagging Speculative Grade Default Rate by Industry	2,0501	1,22E-02	2,2538	6,94E-03	2,1828	1,36E-02
Fama-French Excess Return on Market Factor	1,3814	8,73E-03	1,5085	6,35E-03	1,5468	9,35E-03
Collateral Rank Secured	0,2554	7,21E-03	0,2330	1,25E-02	0,2704	9,36E-04
Tranche Safety Index	0,4548	3,03E-02	0,4339	3,75E-02		
Loss Given Default	0,3273	1,44E-02	0,2751	3,88E-02		
Cumulative Abnormal Returns on Equity Prior to Default	0,3669	1,51E-03	0,3843	1,00E-03	0,4010	9,39E-04
Total Liabilities to Total Assets	0,2653	5,22E-08				
Moody's Original Rating Investment Grade	0,2118	2,80E-02	0,2422	6,84E-03	0,1561	6,25E-02
1-Month Treasury Yield	-0,4298	3,04E-02	-0,3659	1,01E-02	-0,4901	3,36E-02
Size Relative to the Market			0,0366	4,76E-02	0,0648	3,41E-03
Market Value to Book Value			0,1925	2,64E-05	0,1422	5,63E-03
Free-Asset Ratio					-0,2429	2,25E-02
Degrees of Freedom	959		958		783	
Log-Likelihood	-592,30		-594,71		-503,99	
McFadden Pseudo R-Squared (In-Sample)	32,48%		38,80%		41,73%	
McFadden Pseudo R-Squared (Out-Of-Sample) - Bootstrap Mean	21,23%		12,11%		17,77%	
McFadden Pseudo R-Squared (Out-Of-Sample) - Bootstrap Standard Error	2,28%		1,16%		1,70%	

Note:
1 - Annnualized "Return on Defaulted Debt" (RDD) from just after the time of default (1st trading date of debt) until the time of ultimate resolution.

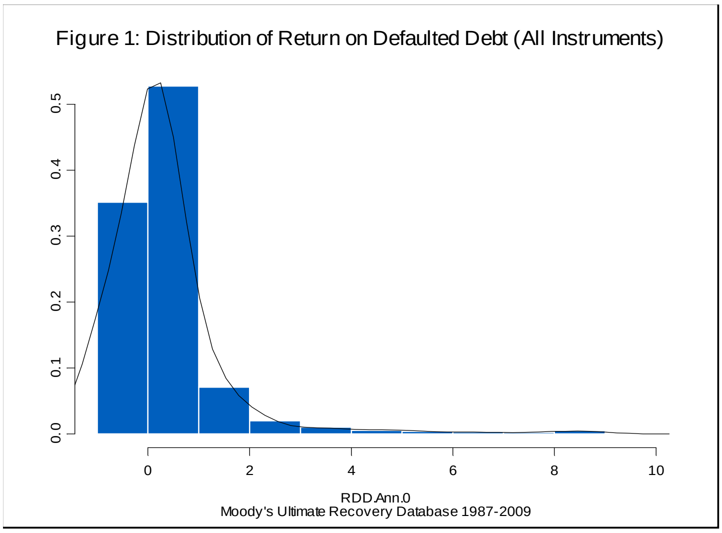

Figure 1: Distribution of Return on Defaulted Debt (All Instruments)

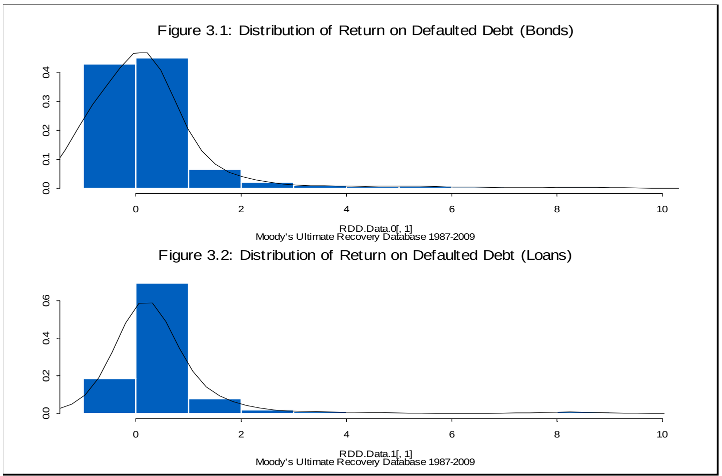

Figure 3.1: Distribution of Return on Defaulted Debt (Bonds)

Figure 3.2: Distribution of Return on Defaulted Debt (Loans)

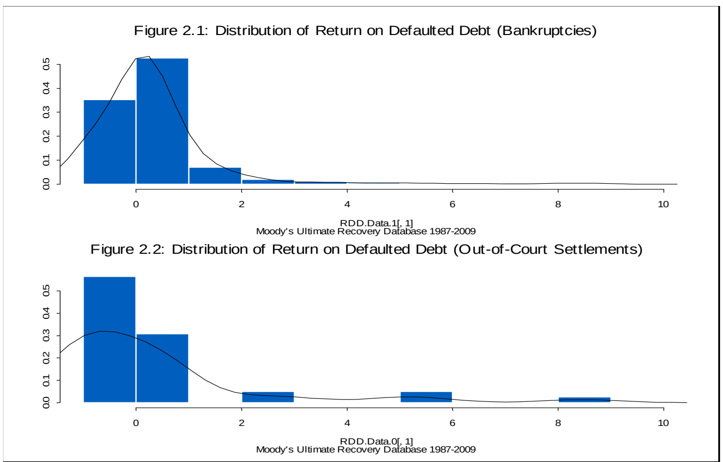

Figure 2.1: Distribution of Return on Defaulted Debt (Bankruptcies)

Figure 2.2: Distribution of Return on Defaulted Debt (Out-of-Court Settlements)