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AN EMPIRICAL STUDY OF EXPOSURE AT DEFAULT

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Abstract

In this study we empirically investigate the determinants of and build a predictive econometric model for exposure at default (EAD) using a sample of Moody's rated defaulted firms having revolving credits. We extend prior empirical work by considering alternative determinants of EAD risk, in addition to the traditional factors (e.g., credit rating.) Various measures of EAD risk are derived and compared. We build a multiple regression model in the generalized linear class and examine the comparative rank ordering and predictive accuracy properties of these. We find weak evidence of counter-cyclicality in EAD. While we find EAD risk to decrease with default risk, utilization has the strongest inverse relation. We also find EAD risk reduced for greater leverage, liquidity, more debt cushion; and increased for greater company size, higher collateral rank or more bank debt in the capital structure of the defaulted obligor. The models are validated rigorously through resampling experiment in a rolling out-of-time and sample experiment. In addition to the credit risk management implications of this study (the parameterization of pricing and portfolio management models), there is use in quantifying EAD risk for banks qualifying for the Advanced IRB approach in the regulatory framework of the Basel II accord.

Keywords: exposure at default, recoveries, default risk, bankruptcy, credit risk, Basel II

JEL Classification: G33, G34, C25, C15, C52

1. Introduction

Committed revolving credit facilities offer borrowers an option to draw funds up to specified limits as dictated by changing circumstances. A great interest in analyzing these stems from the unique characteristics relative to other credit products, which have implications for pricing and credit risk management. Revolving lines appeal to a clientele of borrowers with particular financing strategies and there is an attractive return profile from investor's perspective. There is potential also relief on regulatory capital relative to similar investments. However, these instruments present a great challenge in valuation and risk management, as estimates of revolving credits facility's expected usage at default of the borrower need to be formed. We term this quantity the exposure at default (EAD), or the loan equivalent exposure, and recognize this to be a key parameter in estimating expected loss and credit risk capital for unfunded commitments. A related quantity is the loan equivalency factor (LEQ), which is the proportion of the undrawn commitment that is drawn down upon in the event of default. Financial institutions have a great interest in estimating such quantities from their internal histories, to parameterize credit risk models, as well as to satisfy regulatory requirements. In this study we shall build upon a limited practitioner literature by empirically estimating EADs from publicly available data, and relating these to a set of variables predictive of realized EAD.

2. Review of the Literature

In this Section we review the existing literature on EAD. While for the most part this has been for the most limited to internal bank and trade journal studies that have focused on the level of partial draw-down. There have also been some notable academic studies with a greater focused on pricing of revolving credit, also termed contingent credit lines (CCLs) in some of the literature. Thakor *et al.* (1981) employ an option-theoretic approach to pricing contingent loan commitments. In this setting, the Bank is buys a put option from the obligor and the obligor is sells its debt to the Bank through offering credit lines on pre-specified terms and conditions. The authors proceed to measure the sensitivity of the loan values of such to changes in interest rates and partial draw-downs on a CCL are explained through interest elasticity of demand for borrowed funds and bank customer relationship dynamics. If a firm has infinite opportunities for investment and no restriction on capital structure or leverage, then the interest elasticity will be perfectly elastic, and vice-versa if there are no such opportunities, in which case it will be perfectly inelastic. Alternatively, if looking into the bank customer relationship framework, the later will try to minimize its expected cost of renewal of the line in next year and opportunity for loss by not utilizing the full facility this year when availing the line.

Kaplan, and Zingales (1997) find that the un-drawn portion of credit lines decreases when firms are more liquidity constrained. On the other hand, Gatev, and Strahan (2003) sample 2,695 commitments from the set of all commercial paper backup lines of credit for a large U.S. bank from 1991 to 2002, reporting that partial draw-downs increase when the commercial paper - T bill rate spread rises. Both of these studies indicate presence of a pricing incentive for partial draw-downs, induced by interest rate fluctuations, which subsequently inspired Jones, and Wu (2009) to use the same for modeling framework for partial draw-downs. They model credit quality as a jump-diffusion process, giving rise to partial draw-downs and CCL pricing as a function of the dynamic credit state. The proportion of the credit line drawn is modelled as function of the difference between an alternative opportunity rate and the marginal cost of line borrowings. This opportunity rate is defined as the rate of interest charged to the borrower outside the purview of the defined credit line. In order to incorporate linking of the loan spread to the credit default swap spread of the borrower, the marginal cost of borrowing is defined as function of the reference rate, contractual spread over the reference rate and proportion of the excess that is added to current period loan. Apart from the interest rate differential, the sensitivity of drawdown to the interest rate differential is included to model the amount of partial draw-down. This sensitivity is similar to the interest elasticity proposed by Thakor *et al.* (1981).

Martin, and Santomero (1997) analyze the pricing of CCL from the demand side of firms and show that credit line usage depends on the business growth potential of the firm as well as the uncertainty involved in those investment opportunities. External macro economic variables - such as size of credit line, collateralization, etc. - are also found to be associated with the level of partial draw-downs of obligors on CCLs. Both of these approaches (Thakor *et al.* (1981), and Jones *et al.* (2009)) appear intuitive and convincing, but implementing such in banks where most of the CCL are extended to unrated obligors whose market spread may not be easily available might pose problems of parameterization. Some parameters, such as interest elasticity, may be affected by firm specific behavior as well as present macroeconomic factors. Another approach to estimating usage of limits is in a continuous-time model, where the credit provider and the credit taker interact within a game-theoretic framework, as has been attempted by Leippold *et al.* (2003).

The earliest known study is an analysis conducted by Chase Manhattan Bank (1994), with the assistance of Oliver Wyman Mercer. Drawdowns are studied on 104 revolving credit facilities downgraded in the period 12/91-12/93. The analysis is divided into three parts: 6 month commitments ("short-term"), 1-year commitments ("long-term") and a blend of the two (more specifically, an average for the even years). LEQs are directly estimated for facilities for which defaults are observed in the sample, for speculative ratings (BB and below). However, for investment grade commitments, the migration method is used, which extrapolates factors for better ratings from worse with less time to default using estimated cutback and drawdown rates. Estimated LEQ factors were found to increase with risk rating and tenor.

Attempts to directly estimate partial draw-down has also been undertaken in previous empirical literature. Asarnow *et al.* (1995) analyze utilization patterns, on a monthly basis for revolving commitments in the period 1/87-12/93, for credit lines issued by Citibank to companies having an S&P rating history. They find a downward sloping pattern of usage level from high rated obligors to low rated obligors (i.e., lower rated firms would have already consumed their credit lines earlier than when it approaches default.) However, results by subgrade are not statistically meaningful due to thin data. Utilization rates by rating are computed for 84 months and averaged by rating category. An unpublished version of this study analyzed empirical LEQs based upon subset of 50 facilities downgraded to BB/B or worse in the 1991-1993 period. LEQs are extrapolated to the better risk grades due to lack of investment grade downgrades, and not averaged across facilities, but across quarterly total used and unused for each rating category. Unlike the Chase study, estimated LEQs are found to decrease with increasing credit quality.

Araten *et al.* (2001) document a similar trend of partial draw-downs decreasing as a firm approaches default. They study direct estimates of LEQ factors for 1,021 observations (408 facilities of 399 borrowers) at a quarterly frequency in the period 1Q95-4Q00 for J.P. Morgan Chase borrowers. Given the set of all revolving commitments and advised lines eventually having facility grades rated (accruing) substandard or worse, they track the rating history and usage. The sampling methodology involves stepping back in time from the point of default, calculating the LEQ as change in usage to default relative to unused at a given point in time, at either 1 year intervals or at the time of a grade change. The main result of this analysis is estimated LEQs increasing with time-to-default and with diminished credit risk (i.e., better risk grades.) A pronounced increase in estimated LEQs with tenor was found, with one and five year revolving credits having averages of 32% and 72%, respectively. The decrease in LEQ by grade (worsening credit quality) was found to be milder: 62% for BBB and better, 48% for BBB- to B+, and 27% for B and worse. The overall average is 43.4%, with relatively high 41.4% standard deviation, and a "Barbell" shaped distribution with significant point masses at 0% & 100%. The latter distributional feature is largely an artifact of the truncation of the LEQ estimates, as calculated LEQs greater (less) than 100% (0%) were capped (floored) at 100% (0%). There is a lack of statistical differentiation by other demographics: lending organization, commitment type or size, geography or industry. LEQs are found to decrease with percent usage, but this is highly correlated with risk rating. However, lower LEQs are found for Advised Lines - 17% for one year, but having a similar pattern by grade and time-default. The array of statistical and conceptual issues encountered in this study include outliers, high volatility (on the order of the mean), lack of statistical significance by risk drivers, paucity of data at the investment grade, default definition, sensitivity of estimates to small unused, non-normality, judgmental recoding and data management.

Agarwal *et al.* (2005) document similar patterns to Asarnow *et al.* (1995) and Araten *et al.* examine the utilization of HELC in the U.S. market and confirm that borrowers with deteriorating credit quality increase their utilization. It is argued in several of aforementioned studies that the level of usage is mainly affected by two distinct forces, that the lender might detect deteriorating credit quality of the borrower and cut back the limits (thereby increasing the utilization ratio), or that the borrower may actually use up the line before the lender realizes deteriorating credit quality. As indicated by Qi (2009), examining credit card usage in US, borrowers are more active than lenders in this game of "race to default."

An academic paper that is rather similar to this line of research, Sufi (2005) empirically examines use of bank lines of credit to corporations, using annual 10K filing data. The author finds that the flexibility afforded firms by use of unfunded commitments creates a moral hazard problem, which is mitigated by banks imposing strict covenants, and lending to borrowers with historically high profitability. Another recent manuscript circulating in the regulatory channels, Moral (2006), reviews different methodologies and proposes an optimal framework (from the regulatory viewpoint) for estimating EAD factors. He shows that it is possible to directly impose constraints in the EAD estimation that reflect the preferences of supervisors for not underestimating regulatory capital. It is further shown that a special case of this general problem reduces to a quantile regression of dollar EAD on covariates, which is tractable has several desirable properties.

A recent related study in the option pricing literature, Loukoianova *et al.* (2007), develops a theoretical pricing model for contingent credit lines (CCLs). These are widely used in bank lending and instrumental in the functioning of short-term capital markets, which are closely related to the instruments considered herein. The authors use a financial engineering approach to analyzing the structure of simple CCLs, applying various derivative pricing methods, and discuss issues in the hedging of CCL portfolios.

Jiménez *et al.* (2008) document credit line usage, LEQ factors for revolving commitments, of different firms granted by banks in Spain between 1984 and 2005. They use the Spanish Credit Register, providing a census of all corporate lending within the country over the last twenty years, which is unique in that is includes both defaulting and non-defaulted firms. The final data-set consists of 696,445 credit lines granted to 334,442 firms by 404 banks. The length and breadth of this dataset allows the authors to provide one of the most comprehensive overviews of corporate credit line use and EAD to date. They report a variety of factors such as commitment size, collateralization and maturity of CCLs that affect the usage level. They also report a statistically significant higher usage rate for firms that eventually default at least 3 years prior to default and that the usage monotonically increases as these firms approach default.

Bag, and Jacobs (2010) attempt to build an easy to implement, pragmatic and parsimonious yet accurate model to determine the EAD distribution of a CCL portfolio, modeling each revolver as a portfolio of a large number of option instruments which can be exercised by the borrower, determining the level of usage. Using an algorithm similar to the basic CreditRisk+ and Fourier Transforms, they arrive at a portfolio level probability distribution of usage, and perform a simulation experiment in which we illustrate the convolution of two portfolio segments to derive an EAD distribution.

3. Models for EAD

Let t denote the current time, T a fixed horizon (maturity or ex post calendar time of default) and τ a random time of default. $\mathbf{X}_t$ is a vector of obligor or facility characteristics (e.g., risk rating, product type, financial ratios) observed at time t. Dollar utilization (or used, draw amount) and availability (or commitment, limit) is unused amount at time t is denoted $UTIL_t$ and $AVAIL_t$, respectively. The time t expected dollar exposure at horizon T, conditional upon default occurring before the horizon and upon the vector of covariates $\mathbf{X}_t$, is denoted by $EAD_{\mathbf{X}_t,t,T}$ and satisfies the relations:

$$EAD_{\mathbf{X}_t,t,T} = \mathrm{E}_t\left(UTIL_{\mathbf{X}_t,t} \mid \tau \le T, \mathbf{X}_t\right) = \mathrm{E}_t\left(AVAIL_{X_t,t} \mid \tau \le T, \mathbf{X}_t\right) \quad (1)$$

Note that this assumes that lines are fully drawn at default, or if not that limits are reduced by the bank to the outstanding amount, so that observationally the drawn amount coincides with the lines limit:

$$UTIL_{\mathbf{X}_t,t} = AVAIL_{\mathbf{X}_t,t} \tag{2}$$

Traditionally the dollar EAD at default is estimated through a loan equivalency (LEQ) factor, denoted $LEQ^f_{\mathbf{X}_t,t,T}$, that is applied to the current unused amount $AVAIL_t - UTIL_t$:

$$EAD_{\mathbf{X}_t,t,T} = UTIL_t + LEQ^f_{\mathbf{X}_t,t,T} \times \left(AVAIL_t - UTIL_t \right) \tag{.3}$$

The LEQ factor is the expected portion of the unused drawn down upon in the event of default conditional upon default occurring within horizon and the vector of explanatory variables:

$$LEQ^f_{\mathbf{X}_t,t,T} = \mathrm{E}_t\left(\frac{UTIL_\tau - UTIL_t}{AVAIL_t - UTIL_t} \mid \tau \le T, \mathbf{X}_t \right) \tag{4}$$

Note that by the properties of conditional expectation, substituting (4) in (3) yields (1). The LEQ for a given segment can be estimated by observations of changes in utilization over unused to default:

$$LEQ^f_{\mathbf{X}} = \frac{1}{N_{\mathbf{X}}} \sum_{i=1}^{N_{\mathbf{X}}} \left(\frac{UTIL_{\mathbf{X}_{T_i^D},T_i^D} - UTIL_{\mathbf{X}_{t_i},t_i}}{AVAIL_{\mathbf{X}_{t_i},t_i} - UTIL_{\mathbf{X}_{t_i},t_i}} \right) \tag{5}$$

Where i indexes an observation of a credit line at time t_i, defaulting at time T_i^D, conditional on an vector $\mathbf{X}$ that indexes a segment. Alternatively, we may also think of (5) as a regression of observed LEQ factors in a reference data-set upon a vector of covariates $\mathbf{X}$. An alternative approach estimates an expected *credit conversion factor* (CCF), denoted $CCF^f_{\mathbf{X}_t,t,T}$, that is applied to the current used amount:

$$EAD_{\mathbf{X}_t,t,T} = UTIL_t \times CCF^f_{\mathbf{X}_t,t,T} \tag{6}$$

The CCF is simply the expected gross percent change in the utilized amount between the observation and default date:

$$CCF^f_{\mathbf{X}_t,t,T} = \mathrm{E}_t\left(\frac{AVAIL_\tau}{UTIL_t} \mid \tau \le T, \mathbf{X}_t \right) = \mathrm{E}_t\left(\frac{UTIL\tau}{UTIL_t} \mid \tau \le T, \mathbf{X}_t \right) \tag{7}$$

CCF can be estimated by averaging the observed percent changes in commitment within an EAD segment indexed by $\mathbf{X}$:

$$CCF^f_{\mathbf{X}} = \frac{1}{N_{\mathbf{X}}} \sum_{i=1}^{N_{\mathbf{X}}} \frac{UTIL_{\mathbf{X}_{T_i^D},T_i^D}}{UTIL_{\mathbf{X}_{t_i},t_i}} \tag{8}$$

As with the LEQ factor, this can be thought of in a regression framework. Finally, he expected *exposure-at-default (EAD) factor*, denoted by $EAD^f_{t,T}$, models expected dollar EAD as the expected availability at default:

$$EAD_{\mathbf{X}_t,t,T} = \mathrm{E}_t\left(AVAIL_{\mathbf{X}_t,t} \mid \tau \le T, \mathbf{X}_t\right) = AVAIL_{\mathbf{X}_t,t} \times \mathrm{E}_t\left(\frac{AVAIL_{\mathbf{X}_t,t}}{AVAIL_{\mathbf{X}_t,t}} \mid \tau \le T, \mathbf{X}_t\right) \quad (9)$$

the dollar EAD factor may be factored into the product of the current utilization and an *EAD factor*:

$$EAD_{\mathbf{X}_t,t,T} = AVAIL_{\mathbf{X}_t,t} \times EAD^{f}_{\mathbf{X}_t,T\text{-}t} \quad (10)$$

The EAD factor is the expected gross percent change in availability:

$$EAD^{f}_{\mathbf{X}_t,t,T} = E_t\left(\frac{AVAIL_\tau}{AVAIL_t} \mid \tau \le T, \mathbf{X}_t\right) \quad (11)$$

The EAD factor may be estimated as the average product of the changes in AVAIL from the point of observation to that of default:

$$EAD_{\mathbf{X}} = \frac{1}{N_{\mathbf{X}}}\sum_{i=1}^{N_{\mathbf{X}}}\frac{AVAIL_{\mathbf{X}_{T_i^D},T_i^D}}{AVAIL_{\mathbf{X}_{t_i},t_i}} \quad (12)$$

4. Data and estimation results

We are working with probably the most extensive loss severity database of defaults (bankruptcies and out-of-court settlements) and recoveries, Moody's Ultimate Recovery Database™ (February 2008 release; "MURD"). Most of the issuers in MURD have rated instruments (S&P or Moody's) at some point prior to default, and traded equity, largely representative of the U.S. large corporate loss experience. We have merged MURD with various public sources of information (www.bankruptcydata.com, Edgar SEC filings, LEXIS/NEXIS, Bloomberg, Compustat, and CRSP). The release that forms the basis of our research database contains data on 3,886 defaulted instruments from 1985-2007 for 683 borrowers, for which there is information on all classes of debt in the capital structure at the time of default. All instruments are detailed by facility type, seniority ranking, collateral type, position in the capital structure, original and defaulted amount, resolution outcome, instrument price or value of securities at the resolution of default (emergence from bankruptcy, Chapter 7 liquidation, acquisition or out-of-court settlement). The latter includes either the prices of pre-petition instruments at the time of emergence from bankruptcy or new instruments received in settlement of bankruptcy or other distressed restructuring. In a sub-set of observations, we can obtain the price of traded debt, the equity prices or financial statement data at around the time of default. A smaller sub-set of observations considered in this study consist of revolving loans, for which we can trace the outstanding amounts, limits and ratings in SEC filings (10K and 10Q reports). This subset of MULGD includes 496 obligors, 504 defaults and 544 facilities.

In Tables 1 and 2 we present summary statistics on the EAD risk measures and the key covariates that are the object of this study. First, we discuss the various exposure and EAD risk measures, shown in Table 1. The raw LEQ, having a mean of 63.7%, exhibits extreme variation, ranging in (-21,000%, 106,250%), having a standard deviation (2,759.7%), on the order of100 times the mean. The Winsorized[26] LEQ is little better in terms of stability, a mean of 16.8%, but still varying in a huge domain of -1,165.7% to 804.4%, and standard deviation of 210.4% (about 10 times the mean). The collared LEQ, restricted to the unit interval, averages 42.2%, with standard deviation on the order of the mean of 41.0%. Interestingly, all 3 measures share a median of 33.3%, suggesting that LEQ may be a candidate for the application of robust statistics (e.g., MAD regression). All versions of the LEQ are highly correlated with CCF and EADF, the highest being collared LEQ, with respective rank order correlations of 55.1% and 77.1%. There is an outlier problem with the CCF as well - it averages 1,061.8% and has a maximum of 704,054.4% (note that there is a "natural flooring" at 0 in the case of the CCF factor, as well as with the EADF).

[26] This floors (caps) variable at the 5^{th} (95^{th}) quantile of its ordered value. This is a standard technique for making statistical inference in the presence of extreme outliers and possible contaminated data.

Table 1. Summary Statistics on EAD Risk Measures*

	S&P and Moodys Rated Defaulted Borrowers Revolving Lines of Credits 1985-2007														
	Cnt	Average	Standard Deviation	Minimum	5th Prcntl	25th Prcntl	Median	75th Prcntl	95th Prcntl	Maximum	Skew	Kurtosis	Corr LEQ_{COLL}	Corr CCF_{WIND}	Corr $EADF_{WIND}$
Exposure at Default (EAD)[0]	530	133.140	295.035	158	1.656	20.725	50.000	116.234	508.232	4.250.000	7,5099	82,1857	N/A	N/A	N/A
Dollar Change in Drawn to EAD (DCDE)[1]	2118	48.972	279.972	(3.177.300)	(3.177.300)	(2.056)	7.514	36.617	275.400	4.250.000	6,8444	116,0538	30,07%	35,02%	24,38%
LEQ (Raw)[2]	1582	63,72%	2759,66%	-21000,00%	-21000,00%	-12,75%	33,28%	87,64%	231,76%	106250,00%	35,7617	1391,0651	9,47%	15,34%	11,49%
LEQ (Collared)[3]	1582	42,21%	40,92%	0,00%	0,00%	0,00%	33,28%	87,64%	100,00%	100,00%	0,3054	-1,5700	100,00%	55,06%	77,09%
LEQ (Winsorized)[4]	1582	16,80%	210,38%	-1165,74%	-1165,74%	-12,75%	33,28%	87,64%	231,76%	804,43%	-1,9084	13,5038	58,59%	35,70%	53,93%
CCF[5]	1330	1061,8%	20032,7%	0,47%	0,47%	85,30%	111,11%	198,86%	860,29%	704054,38%	32,9416	1145,3158	1,89%	15,37%	-2,10%
CCF (Winsorized)	1330	190,4%	203,4%	26,29%	26,29%	85,30%	111,11%	198,86%	855,66%	860,29%	2,27	4,45	55,06%	100,00%	38,44%
EAD Factor[6]	1587	143,40%	2666,07%	0,37%	0,37%	42,46%	70,67%	95,96%	152,86%	106250,00%	39,80	1584,89	4,95%	41,74%	7,48%
EAD Factor (Winsorized)	1587	70,76%	36,94%	11,24%	11,24%	42,46%	70,67%	95,96%	152,39%	152,86%	0,29	-0,39	77,09%	38,44%	100,00%
Utilization[7]	1621	45,85%	32,85%	0,00%	0,00%	14,00%	48,04%	74,27%	95,00%	100,00%	-0,06	-1,35	-33,50%	-61,58%	1,03%
Commitment[8]	1621	184.027	383.442	217	217	40.000	80.000	176.400	570.000	4.250.000	6,24	48,28	2,51%	-4,41%	-6,88%
Drawndown Rate[9]	879	0,39%	7,00%	-0,10%	-0,10%	-0,02%	0,01%	0,05%	0,41%	181,97%	23,17	561,82	-4,38%	-2,80%	-2,76%
Cutback Rate[10]	1126	88,50%	2791,11%	-96,07%	-96,07%	0,00%	0,00%	0,00%	66,67%	93650,00%	33,54	1125,34	4,51%	1,52%	3,60%
Drawn[11]	1621	71.576	163.029	0	0	5.557	26.463	76.900	260.000	3.090.000	8,41	107,87	-14,69%	-18,58%	-5,85%
Undrawn[12]	773	112.450	329.695	0	0	13.082	34.099	82.300	396.500	4.250.000	7,79	73,49	9,54%	12,53%	-5,08%

496 (504) defaulted borrowers (instances of default), having 544 revolving credit exposures and sampled prior to default at one year anniversarries, changes in risk rating prior or other signifcant events prior to default

0 - Dollar EAD (or outstanding amount at default)
1 - Change from outstanding amount at an observation date to dollar EAD
2 - Empirically measured Loan Equivalent Exposure where $LEQ_{t,T}$ = ($Drawn_T$ - $Drawn_t$)/$Undrawn_t$, T (t) = default (observation) date
3 - LEQ floored (capped) at 0% (100%) = max(min(LEQ,1),0)
4 - LEQ floored (capped) at the 1st (99th) percentiles = max(min(LEQ,516.95%),-874.57%)
5 - Credit Conversion Factor: $CCF_{t,T}$ = $Drawn_T$/$Drawn_t$, T (t) = default (observation) date.
6 -Exposure at Default Factor: $EAD_{t,T}$ = $Exposure_T$/$Exposure_t$, T (t) = default (observation) date.
7- $Utilization_t$ = $Drawn_t$/$Commitment_t$ where $Commitment_t$ = $Drawn_t$ + $Undrawn_t$
8 - $Commitment_t$ = Total legal commitment or limit on credit line at observation date t ($000s)
9 - Percent change in drawn from date prior until observation date
10 - Percent change in commitment from date prior until observation date
11 - $Drawn_t$ = Total amount outstanding on line at time t
12 - $Undrawn_t$ = Total undrawn commitment (legal commitment minus drawn) on line at time t

Source: Moody's Rated Defaulted Borrowers Revolving Lines of Credits (1985-2007)

The Winsorized version is more reasonable, an average of 190.4%, showing that by this measure, lines on average are 90% higher at default as compared to earlier points in time. The average EADF of 143.4% suggests that typically *increase* about 43% leading to default; however, average Winsorized EADF of 70.8% suggests that typically lines are reduced by about 30% leading to default. Note that there are slightly different counts for these (1582, 1330 and 1587 for LEQ, CCF and EADF, respectively), as there are different data requirements for each (i.e., non-zero unused and used for LEQ and CCF, respectively). The severity of the outlier problem associated with the LEQ factor can be more easily seen through visual inspection of the raw, Winsorized and collared distributions. Both the raw and Winsorized LEQ distributions are extremely heavy tailed and skewed, more like something more out of the Stable family (possibly with undefined 1st and 2nd moments) than a normal or even spherical distribution. On the other hand, the collaring of LEQ in Figure 1.3 yields a bimodal distribution, having point masses at 0 and 1, and approximately uniform between these boundaries. This appears to be potentially well approximated by a beta distribution, and resembles empirical distributions of ultimate Loss-Given-Default (LGD) observed in the literature; see Araten *et al.* (2004b) and Jacobs *et al.* (2007).

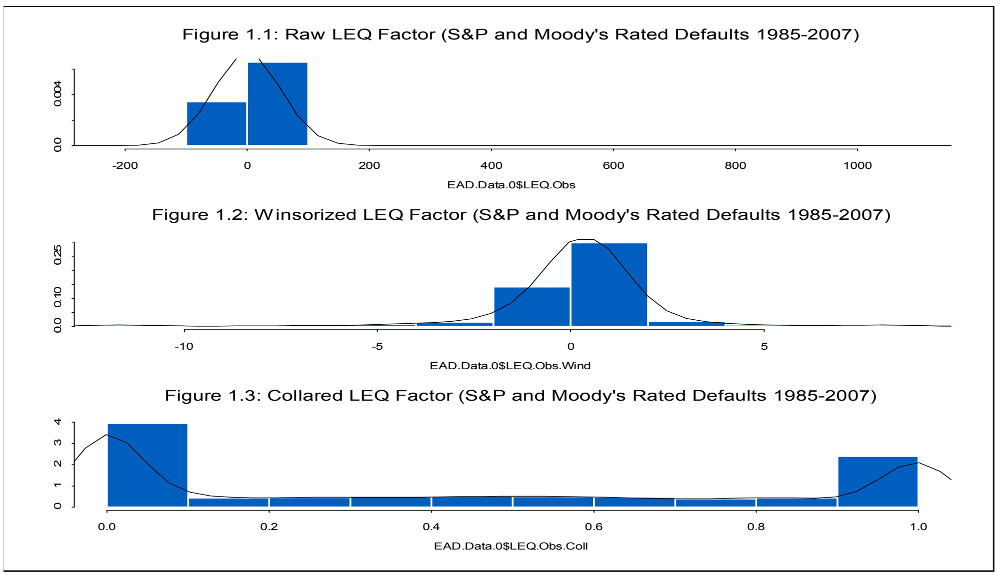

Figure 1. Distributions of LEQ Factors

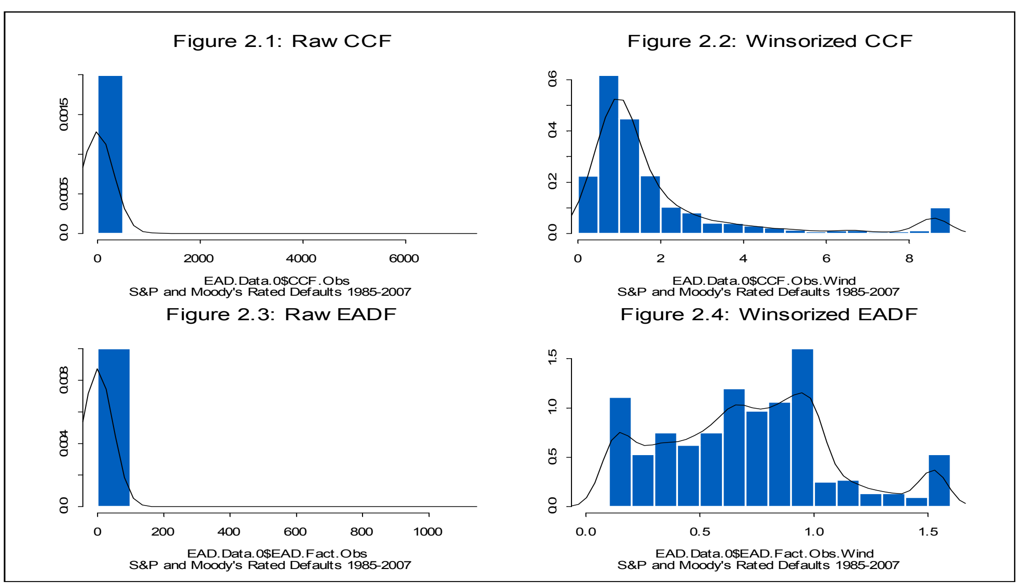

Figure 2. Distributions of CCF and EADF factors

Table 2. Summary statistics: borrower, facility and market characteristics

S&P and Moodys Rated Defaulted Borrowers Revolving Lines of Credits 1985-2007													
	Cnt	Average	Standard Deviation	Minimum	25th Prcntl	Median	75th Prcntl	Maximum	Skew	Kurt	Corr LEQ_{COLL}	Corr CCF_{WIND}	Corr $EADF_{WIND}$
Time-to-Defaul[2]	1616	1,7776	1,3167	-0,1644	0,7671	1,4986	2,7171	6,4192	0,85	-0,07	15,09%	18,14%	0,27%
Obligor Rating[3]	622	2,9873	0,8672	1,0000	3,0000	3,0000	3,0000	5,0000	-0,45	0,51	-17,80%	-16,07%	-11,28%
Leverage 1 [4]	537	74,95%	21,88%	6,05%	63,82%	81,90%	93,04%	100,00%	-1,06	0,26	-5,48%	-10,20%	2,29%
Leverage 2 [5]	722	97,35%	37,60%	17,85%	76,08%	91,55%	106,18%	411,19%	2,49	11,77	-6,62%	4,43%	-5,48%
Size[6]	725	277,46%	50,77%	43,96%	242,36%	275,88%	308,26%	501,67%	0,48	2,30	17,80%	5,33%	7,37%
Intangibility[7]	474	35,70%	36,69%	0,00%	0,00%	25,93%	64,81%	131,79%	0,76	-0,53	12,61%	3,68%	10,23%
Liquidity[8]	685	152,96%	99,00%	6,06%	92,30%	139,77%	198,79%	1255,70%	2,88	23,36	-9,18%	-8,79%	-8,95%
Cash Flow[9]	672	-235,64%	10002,79%	-43416,47%	-15,84%	2,22%	357,75%	173952,22%	8,55	157,51	5,40%	1,96%	5,90%
Profitabilty[10]	721	-2022,71%	35498,24%	-673548,90%	-24,29%	-4,58%	-0,17%	81,16%	-18,86	355,70	-7,77%	-10,45%	-4,53%
Discounted Ultimate LGD[11]	707	7,76%	29,76%	-90,12%	-5,73%	0,00%	6,24%	100,00%	1,07	1,85	10,02%	10,13%	9,29%
Market Implied LGD at Default[12]	175	31,16%	23,48%	-3,72%	10,25%	28,00%	49,63%	90,00%	0,51	-0,68	12,33%	16,48%	9,44%
Creditor Rank[13]	1621	1,3967	0,7495	1,0000	1,0000	1,0000	2,0000	6,0000	2,38	6,80	7,06%	9,03%	2,00%
Collateral Rank[14]	1621	3,2529	1,4428	1,0000	3,0000	3,0000	3,0000	8,0000	2,16	4,64	15,94%	12,57%	11,85%
Percent Debt Cushion[15]	1621	25,70%	32,51%	0,00%	0,00%	0,00%	52,00%	99,48%	0,81	-0,84	-15,27%	-10,35%	-4,52%
Speculative Grade Default Rate[16]	1621	5,67%	2,92%	0,00%	3,15%	6,03%	7,05%	13,26%	0,44	-0,50	-9,09%	-9,53%	-9,31%
Speculative Grade Default Rate - Industry [17]	1621	5,91%	4,11%	0,00%	2,96%	5,13%	7,95%	20,00%	0,77	0,11	-7,55%	-7,44%	-7,71%
Risk-Free Return[18]	1621	0,40%	0,14%	0,06%	0,35%	0,43%	0,50%	0,72%	-0,78	0,18	0,10%	2,67%	0,72%
Excess Equity Market Return[19]	1621	0,52%	4,46%	-10,76%	-0,46%	1,50%	3,41%	8,00%	-1,09	0,83	4,22%	5,85%	3,05%
Equity Market Size Factor (Fama-French)[20]	1621	0,26%	2,76%	-5,74%	-1,64%	0,44%	1,52%	8,43%	0,34	0,40	-1,22%	0,39%	-2,06%
Equity Market Value Factor (Fama-French)[21]	1621	2,08%	4,59%	-5,68%	-0,74%	1,67%	4,23%	13,80%	0,58	0,43	-1,58%	-4,38%	-4,63%
Cumulative Abnormal Equity Return[22]	525	-5,99%	66,63%	-152,71%	-51,63%	-6,96%	36,32%	174,70%	0,31	-0,13	-7,14%	-9,38%	-4,11%
Number of Creditor Classes[23]	1621	2,3307	0,8228	1,0000	2,0000	2,0000	3,0000	6,0000	0,91	1,51	0,72%	-2,51%	-2,52%
Percent Secured Debt[24]	1621	47,76%	31,25%	0,00%	23,54%	43,42%	70,04%	113,82%	0,32	-0,96	17,35%	2,55%	14,67%
Percent Subordinateded Debt[25]	1621	28,93%	33,28%	0,00%	0,00%	13,29%	50,11%	111,79%	0,90	-0,51	-4,10%	-2,19%	-4,38%
Percent Bank Debt[26]	1621	44,81%	28,98%	0,00%	22,20%	41,17%	62,60%	113,82%	0,50	-0,66	13,55%	8,50%	18,92%

496 (504) defaulted borrowers (instances of default), having 544 revolving credit exposures and sampled prior to default at one year anniversaries, changes in risk rating prior or other significant events prior to default

2 - Time from observation date to default of revolving credit (years)
3 - Numeric codes for major ratings (on S&P scale): 1 =AAA- BBB, 2 = BB, 3 = B, 4 = CCC-CC, 5 = C
4 - Leverage measured by the ratio of long term debt to the market value of equity observed at 1 and 2 years prior to default
5 - Leverage measured by the ratio of total debt to the book value of assets observed at 1 and 2 years prior to default
6 - Company size measured by the logarithm of the book value of assets observed at 1 and 2 years prior to default
7 - Tangibility of assets measured by the ratio of intangible to total assets observed at 1 and 2 years prior to default
8 - Liquidity measured by the current ratio (current assets to current liabilities) observed at 1 and 2 years prior to default
9 - Cash flow measured by the ratio of free cash flow to total sssets observed at 1 and 2 years prior to default
10 - Profitability measured by the profit margin (ratio of net income to net sales) observed at 1 and 2 years prior to default
11 - Economic Loss Given Default measured as the complement of the ratio of the value of securities receved in settlement of default to the outstanding due at default
12 - Economic Loss Given Default measured as the complement of the ratio of the market value defaulted debt shortly after (30-45 days) the event of default to the outstanding due at default
13 - Rank of major creditor class in bankruptcy or renegotiation to which the defaulted revolver belongs
14 - Ranking of collateral quality.
15 - Proportion of debt in the capital structure subordinated to the instrument.
16 - Moody's trailing 1-year default rate calculated for quarterly cohorts of speculative grade rated issuers
17 - Moody's industry specific (broad categories) trailing 1-year default rate calculated for quarterly cohorts of speculative grade rated issuers
18 - Return on 1-Month Treasury Bills (constant maturity)
19 - The excess return on the market measured as the value-weight return on all NYSE, AMEX, and NASDAQ stocks (from CRSP) minus the one-month Treasury bill rate (from Ibbotson Associates)
20 - SMB (Small Minus Big) is the average return on the three small portfolios minus the average return on the three big portfolios, SMB = 1/3 (Small Value + Small Neutral + Small Growth) - 1/3 (Big Value + Big Neutral + Big Growth)
21 - HML (High Minus Low) is the average return on the two value portfolios minus the average return on the two growth portfolios HML =1/2 (Small Value + Big Value)- 1/2 (Small Growth + Big Growth)
22 - Cumulative excess return on equity of issuer 3 months prior to measurment date
23 - Number of major creditor classes in the capital structure
24 -Proportion of secured debt in the capital structure
25 -Proportion of bank debt in the capital structure

Source: Moody's Rated Defaulted Borrowers Revolving Lines of Credits (1985-2007)

We first turn our attention to features of the data that characterize the pattern of usages and limits. Average percent utilization ("UTIL") in the data-set is 45.9%, with a median of 48.0%, and a standard deviation of 32.9%. UTIL is inversely correlated with two of the EAD risk measures, Spearman correlation coefficients of -33.5% and -61.6% for LEQ and CCF, respectively; however, it has negligible correlation with EADF of 1.0%. The average undrawn amount ("UNDRN") is $112.5 Million, and has positive Spearman rank correlations the same two EAD risk measures as well, coefficients of 9.5% and 12.5% for LEQ and CCF, respectively; however, it has negative correlation with EADF of -5.1%. The drawn amount ("DRAWN"), averaging $71.6 Million is most strongly inversely related to LEQ and CCF, respective correlation of -14.7% and -18.6%, and a little less so for EADF (-5.9%). The average *cutback rate* ("CR"), or percentage change in commitment across subsequent observation dates (a measure of the speed with which banks clamp down on commitments as borrowers approach default), exhibits massive upward skew with an average of 88.5%, median of 0.0% and maximum of 93,650.0%. The CR exhibits a mild positive relationship with the LEQ and EADF, respective correlations of 4.5% and 3.6% with those variables, and less so but still positive with respect to CCF, a coefficient of 1.5%. On the other hand, the *drawdown rate* ("DR"), a measure of the aggressiveness with which obligors tap lines on their way to default, exhibits a more stable looking yet positively skewed distribution, averaging 0.4% and ranging in -0.1% to 182.0%. Perhaps counter-intuitively, correlations with all EAD risk parameters are negative but of mild magnitude (-4.4%, -2.8% and -2.8% with LEQ, CCF and EADF, respectively).

Turning now to Table 2, let us first consider time-to-default ("TTD") and obligor risk rating ("ORR") variables, both of which having played a central role in the Citibank and Chase studies[27]. While both the average and median TTDs are a little over a year and a half, 1.50 and 1.78 years, respectively, this distribution has an elongated right tail, evidenced by a maximum of 6.42 years and a standard deviation on the order of the mean of 1.32 years (somewhat like an exponential distribution). In line with the hypothesis that at greater horizons to default borrowers have an augmented opportunity to draw down upon unfunded commitments, Spearman correlations are positive and significant for 2 of the risk parameters, robust for CCF (8.1%) and LEQ (15.1%); but of questionable magnitude for EADF (0.3%). ORRs are grouped into 5 major classes, and to these are attached numerical codes as follows: 1 = AAA-BBB, 2 = BB, 3 = B, 4 = CC-CCC and 5 = D.[28] Average numerical code rating is 2.98, with a median of 3, indicative of the high yield nature of this data-set. Consistent with evidence from bank studies, and in line with the story that these proxy for the level of controls over borrowers in restricting their draw downs in the event of credit deterioration, correlations are moderately negative (-17.8%, -16.1% and -11.3% for LEQ, CCF and EADF, respectively). An alternative measure of obligor quality is the Cumulative Abnormal Return ("CAR") on equity in the month leading up to default. For the 525 observations for which traded equity returns are available, we find that average CARs are negative but moderate in magnitude, -6.0%; however, there is a mild positive skew to these, as the median is -7.0% and maximum is 174.7%. These are negatively and correlated to EAD risk measures, moderate magnitudes of -7.1%, -9.4% and -4.1% for LEQ, CCF and EADF, respectively. To the extent that higher rated obligors may have less negative CARs prior to default, this is in line the explanation or stylized fact of inverse correlation of EAD risk measures with measures of obligor default probability.

Turning to the obligor financial characteristics in Table 2, measured at 1 year or 2 prior to default, we can see that there is some evidence that financial state has an influence on EAD risk measures. Leverage, as measured by the ratio of long-term debt to the market value of equity ("LTD/MVE"), averaging 75.0% in this sample, is negatively correlated with of two of the three EAD risk measures, having rank correlations of -5.5% and -10.0% for LEQ and CCF, respectively, but a positive and nearly negligible correlation of 2.3% with EADF. This result carries over to the regression models for LEQ and CCF, but not EADF, where another measure enters, the ratio of total debt to the book value of equity ("TD/BVE"), averaging 97.4% in this sample. TD/BVE is negatively correlated two of the three EAD risk measures, having rank correlations of -6.6% and -5.5% for LEQ and EADF, respectively, but a positive and moderate correlation of 4.4% with CCF. This may strike some as counterintuitive, but perhaps the story is that more highly levered companies have less capacity to draw down additionally on the way to distress. However, this may be contrary to evidence that leverage is *inversely* related to ultimate LGD (Jacobs *et al*. 2007), if one believes that LGD and EAD are inversely correlated (i.e., if creditors

[27] One may argue that TTD should not be used as an explanatory variable, as it is not known at the time of assigning an EAD measure. Usually banks focus on a one year horizon. However, in dealing with longer term revolvers, this may be a useful variable to track. However, there are methods of incorporating this into forward looking of EAD risk measures – see Jacobs (2001) for a technique utilizing the term structure of MKMV EDF™ estimates.

[28] In some cases this is just short of default in cases, where it was verified that the rating agencies assigned a D grade, but the earliest event of default had not yet occurred.

induce an earlier default prior to further deterioration, EAD may be higher as there is less opportunity to work the exposure down). However, to the extent that this means the default is in some sense more predictable (i.e., the borrower is already closer to the default point), this may mean that prior to default there is a greater intensity of monitoring, and hence a reduction in EAD risk.

Second, company size appears to be directly related to EAD risk metrics, as the logarithm of the Book Value of Total Assets ("BVTA"), averaging 2.77, has positive rank correlations of reasonable magnitudes with these (17.8%, 5.3% and 7.4% with LEQ, CCF and EADF, respectively). This result carries over to all of the regression models. To the extent that larger companies may be under less scrutiny, or be subject to less restrictive covenants, this is a reasonable result. Intangibility of assets, as measured by the ratio of intangibles to total book value of assets ("INTA"), averages 35.7% in the sample. INTA has a positive relationship with some EAD risk measures, sizable correlations of 12.6% and 10.2% with LEQ and CCF, respectively, but only a small correlation of 3.7% with CCF. This result holds up in 2 of the 3 the multiple regression analysis, in the case of LEQ and EADF, whereas this dimension does not enter the model for CCF. On the other hand, a robust result is that more liquid companies have lower measures of EAD risk. The ratio of current assets to current liabilities (the *current ratio* or "CR"), having a mean of -153.0% in the sample, possesses significant inverse correlations with EAD risk parameters of -9.2%, -8.8% and -10.0% for LEQ, CCF and EADF, respectively. This result carries over to all of the multiple regression models. The relationship between EAD risk measures and cash flow, as measured by the ratio of ratio of free assets to the book value of total assets ("FA/BVTA"), with an average of -235.6%, is not as robust. FA/BVTA has only modest positive correlations with EAD risk measures: 5.4%, 2.0% and 5.9% with respect to LEQ, CCF and EADF, respectively. Indeed, this variable does not show up in any of the multiple regression models. The correlations are all modest for a measure of profitability, as measured by the Profit Margin ("PM"), having a median (mean) in this sample of 2.2% (-2022.7%), which is an extremely negatively skewed variable. PM has consistently negative correlations across EAD risk measures: -7.8%, -10.5% and -4.5% for LEQ, CCF and EADF, respectively; further, this result holds in all the multiple regression models considered in this paper.

The next set of variables that we consider concern the structure of the instrument - the Creditor Rank ("CRED"), Collateral Rank ("COLL") and the Percent Debt Cushion ("CUSH"). The vast majority of these revolvers are at the top of the capital structure, as the median of CRED is 1, so a priori we may think that this variable has limited efficacy in explaining EAD risk. But in fact, the correlations are positive and significant in 2 out of 3 cases: 7.1%, 9.0% and 2.0% for LEQ, CCF and EADF, respectively. However, this variable enters none of the multiple regression models. Similarly, the collateral on these loans tends to be the best ranked, the 75th percentile of COLL being 3 out of 8. Here correlations are also positive, but a larger than for CRED and sizable for all EAD risk measures: 15.9%, 12.6% and 11.9% for LEQ, CCF and EADF, respectively. And this result holds up in the multiple regression models. At first it may strikes us as puzzling that better collateral might be associated with higher EAD, as we might think that banks could more easily liquidate such and reduce their exposure, but one may also argue that such loans are under a lower degree of scrutiny. Finally, the mean CUSH on these revolvers is 25.7%, indicating that they tend to be in the middle tranches of the capital structure, despite being secured by high quality collateral or being in the highest creditor class. However, unlike the other facility structure characteristics, greater CUSH (a superior structural characteristic) is significantly (at least for 2 out of 3 measures) associated with lower measures of EAD risk: respective correlations of -15.3%, -10.4% and -4.5% for LEQ, CCF and EADF. This variable significantly enters all of the multiple regression models with a negative sign.

Turning to the downturn characterization of EAD risk, or relation to systematic (or aggregate equity market) factors, we some evidence of counter-cyclicality in examining the univariate correlations with aggregate default rates. The Moody's Speculative 12-Month Trailing Default Rate ("MSG12MTDR"), averaging 5.7% over the sample period, exhibits moderate negative correlation with our EAD risk measures (-9.1%, -9.5% and -9.3% for LEQ, CCF and EADF, respectively). The results are qualitatively similar but slightly weaker for the MSG12MTDR by industry group, averaging 5.9% in the sample: -7.6%, -7.4% and 7.7% for LEQ, CCF and EADF, respectively. However, the correlations with the excess return on a broad equity index return, the Fama-French Market Factor ("FF-MKT"), (averaging 0.62% in the sample, are smaller but still showing counter-cyclicality: correlations of 4.2%, 5.9% and 3.1% for LEQ, CCF and EADF, respectively. Regarding the two other Fama-French factors, the size ("small minus big" or "FF-SMB") and value ("high minus low" or "FF-HML") portfolio returns, we don't see much of a relationship: correlations -1.2%, 0.4% and -2.1% (-1.6%, -4.4% and -4.6%) in the case of FF-SMB (FF-HML) for LEQ, CCF and EADF, respectively. None of the Fama-French factors enters the multiple regression models. Finally for systematic risk variables, we consider a short-term interest rate, the 1-Month Treasury Bill Yield ("1M-TBY"), which seems to have little relation to EAD risk measures: correlations of

0.1%, 2.7% and 0.7% for LEQ, CCF and EADF, respectively; and this variable enters non of our final regression models.

Finally, let us consider some capital structure variables. Loans in this data-set tend to be associated with relatively simple structures, as the median number of major creditor classes ("NUMCL") is 2. This variable is inversely, albeit weakly, with EAD risk measures for two of the EAD risk measures, correlations of 0.7%, -2.5% and -2.5% for LEQ, CCF and EADF, respectively. This is perhaps counterintuitive, as it may be natural to think that monitoring would be facilitated in simpler capital structures, hence a positive relationship would obtain. However, this variable is not significant in any of the multiple regression models, as indicated by the insignificance of the univariate correlations. Second, we see that on average 47.8% of the debt on the obligor's capital structure in the data-set is secured ("PERCSEC"), and a greater proportion of such is associated with higher measures of EAD risk, correlations of 17.4%, 2.6% and 14.7% for LEQ, CCF and EADF, respectively; and these correlations are substantial, for LEQ and EADF, but not for CCF. Again, as we might be tempted to think that when more debt is secured controls are augmented, and hence drawdowns are limited, the data does not bear out such intuition. However, unlike as with the number of major creditor classes, this variable does make it into the multivariate analysis for two of the EAD risk measures, LEQ and EADF. But, a variable that carries over to the regressions for all three measures is the proportion of bank debt ("PERCBNK") in the capital structure, which averages 44.8% in the sample. This is positively associated with measures of EAD risk, having substantial correlations of 13.6%, 8.5% and 18.9% for LEQ, CCF and EADF, respectively (albeit the relationship for the CCF factor is weaker as compared with the other two on a univariate basis). Again, we have a result that is at first blush at odds with intuition. However, there is a story that one can tell: to the extent that banks may be successful at accelerating defaults (due to information asymmetry or comparative advantage in monitoring ability), thereby lowering there loss severity (Carey *et al*. 2007, and Jacobs, 2007), we might expect this to augment EAD if we also believe that EAD and LGD are inversely related. Alternatively, this variable might be correlated with the presence of a larger banking syndicate, which could imply coordination difficulties, less effective monitoring and hence higher measures of EAD risk.

In Table 3 we analyze the LEQ factors by credit rating and time-to-default, in a fashion similar to that of Araten *et al.* (2001), where we create the buckets by rounding to the nearest year[29], showing collared LEQ along with Winsorized CCF and EADF. Note that we only have observations for which the obligor is rated for 1,416 of the 1,582 measured LEQ factors. Furthermore, the population is largely sub-investment grade, as only 110 observations are in the AAA-BBB class. We observe that the collared LEQ decreases almost monotonically with worsening risk rating, in line with previous evidence: averages of 69.1%, 40.8%, 42.8%, 36.9% and 20.2% for ratings AAA-BBB, BB, B, CC-CCC and C, respectively. However, the while there is a general pattern of decrease with increasing credit risk in the Winsorized and raw versions of the LEQ factor, it is not monotonic. Even looking at broader groupings for those (not shown in the table), monotonicity also fails to obtain, as we have averages of -0.004% and 0.006% for BB-B and CCC-C. Nevertheless, with all three measures LEQ we do observe a significant rank correlation with the obligor rating, and for all three Kruskall-Wallis (KW) tests (not reported) of the median LEQs in each rating reveal that we can reject the null hypothesis that LEQ is not different across rating classes. Averages of the LEQs for the NRs show them to be between the B and CCC-CC, a mean of 38.4%.

[29] Results (available upon request) are very similar following the methodology of Araten *et al.* (2001), which is the "round up technique", such that we bucket by the next highest integer number of years (i.e., a time to default of 1.3 years is in the 2 year bucket).

Table 3. Estimated Loan Equivalency Factors by Rating and Time-to-Default[1]

Collared LEQ

Count (Time-to-Default (yrs))

Risk Rating	<1	1	2	3	4	5	>5	Total
AAA-BBB	11	43	25	17	10	4	0	110
BB	13	59	43	29	16	15	0	175
B	103	254	194	115	76	48	3	793
CCC-CC	84	102	61	30	16	8	0	301
C	17	8	4	5	3	0	0	37
NR	35	60	42	19	7	3	0	166
Total	263	526	369	215	128	78	3	1582

Average (Time-to-Default (yrs))

Risk Rating	<1	1	2	3	4	5	>5	Total
AAA-BBB	43.44%	64.56%	65.26%	84.93%	92.86%	84.58%	0.00%	69.06%
BB	27.82%	38.90%	42.13%	45.91%	43.91%	42.35%	0.00%	40.79%
B	33.14%	41.51%	43.92%	42.60%	52.77%	49.94%	14.00%	42.66%
CCC-CC	22.29%	32.97%	47.38%	54.80%	55.05%	55.30%	0.00%	36.85%
C	9.91%	28.21%	9.71%	47.64%	25.67%	0.00%	0.00%	20.22%
NR	33.17%	37.73%	39.79%	37.88%	44.61%	82.39%	0.00%	38.40%
Total	28.35%	40.81%	44.89%	47.79%	54.00%	52.05%	14.00%	42.21%

Standard Deviation (Time-to-Default (yrs))

Risk Rating	<1	1	2	3	4	5	>5	Total
AAA-BBB	45.75%	38.08%	40.54%	27.94%	12.39%	19.09%	N/A	37.78%
BB	38.00%	39.32%	41.45%	42.87%	44.64%	38.14%	N/A	40.42%
B	40.97%	39.61%	37.79%	38.43%	42.18%	40.63%	16.37%	39.67%
CCC-CC	37.58%	39.91%	40.05%	41.41%	44.04%	48.67%	N/A	41.37%
C	28.43%	44.72%	14.10%	24.78%	23.10%	N/A	N/A	32.34%
NR	46.50%	43.02%	41.09%	40.79%	41.57%	30.51%	N/A	42.73%
Total	40.40%	40.58%	39.37%	40.12%	42.10%	40.48%	16.37%	40.92%

Winsorized CCF

Count (Time-to-Default (yrs))

Risk Rating	<1	1	2	3	4	5	>5	Total
AAA-BBB	9	26	19	11	5	0	N/A	70
BB	12	54	32	23	14	11	N/A	146
B	97	229	155	88	53	24	1	647
CCC-CC	80	89	46	18	11	4	N/A	248
C	16	6	4	4	3	N/A	N/A	33
NR	38	69	43	22	7	2	N/A	186
Total	252	473	299	166	93	41	1	1330

Average (Time-to-Default (yrs))

Risk Rating	<1	1	2	3	4	5	>5	Total
AAA-BBB	205.22%	286.37%	322.50%	537.29%	448.88%	N/A	N/A	336.78%
BB	207.52%	142.12%	155.31%	180.38%	211.69%	298.37%	N/A	174.86%
B	131.68%	175.62%	211.32%	215.59%	257.96%	223.89%	78.91%	191.41%
CCC-CC	122.80%	163.94%	175.76%	220.39%	201.72%	135.60%	N/A	158.18%
C	92.74%	104.38%	115.99%	182.23%	171.10%	N/A	N/A	115.64%
NR	195.10%	171.47%	170.50%	273.69%	421.22%	652.00%	N/A	200.41%
Total	142.19%	174.17%	199.78%	239.45%	264.09%	256.14%	78.91%	190.42%

Standard Deviation (Time-to-Default (yrs))

Risk Rating	<1	1	2	3	4	5	>5	Total
AAA-BBB	196.36%	258.07%	287.26%	323.32%	376.79%	N/A	N/A	290.00%
BB	232.87%	157.90%	167.99%	199.22%	236.76%	302.28%	N/A	196.15%
B	116.77%	186.99%	218.97%	225.87%	249.85%	230.64%	N/A	202.01%
CCC-CC	130.00%	184.42%	132.15%	215.32%	243.75%	142.16%	N/A	165.52%
C	15.29%	24.51%	20.38%	24.92%	69.04%	N/A	N/A	41.56%
NR	216.33%	175.71%	176.32%	260.61%	348.42%	294.57%	N/A	213.23%
Total	148.80%	186.84%	203.15%	242.77%	262.25%	259.84%	N/A	203.38%

Winsorized EADF

Count (Time-to-Default (yrs))

Risk Rating	<1	1	2	3	4	5	>5	Total
AAA-BBB	11	43	25	17	10	4	N/A	110
BB	13	59	43	29	17	15	N/A	176
B	104	254	194	115	76	48	3	794
CCC-CC	85	103	62	30	16	8	N/A	304
C	17	8	4	5	3	N/A	N/A	37
NR	35	60	42	19	7	3	N/A	166
Total	265	527	370	215	129	78	3	1587

Average (Time-to-Default (yrs))

Risk Rating	<1	1	2	3	4	5	>5	Total
AAA-BBB	80.40%	78.91%	87.83%	110.46%	120.18%	97.80%	N/A	90.40%
BB	68.43%	66.17%	63.21%	67.02%	69.37%	61.60%	N/A	65.67%
B	72.80%	71.75%	69.02%	63.75%	73.49%	70.80%	26.39%	70.00%
CCC-CC	68.84%	64.57%	65.95%	70.72%	69.74%	76.25%	N/A	67.23%
C	62.46%	68.86%	58.99%	59.38%	47.74%	N/A	N/A	61.86%
NR	77.45%	72.86%	77.48%	67.59%	75.42%	113.92%	N/A	75.24%
Total	71.58%	70.39%	69.95%	69.10%	75.61%	72.63%	26.39%	70.76%

Standard Deviation (Time-to-Default (yrs))

Risk Rating	<1	1	2	3	4	5	>5	Total
AAA-BBB	80.40%	78.91%	87.83%	110.46%	120.18%	97.80%	N/A	90.40%
BB	68.43%	66.17%	63.21%	67.02%	69.37%	61.60%	N/A	65.67%
B	72.80%	71.75%	69.02%	63.75%	73.49%	70.80%	26.39%	70.00%
CCC-CC	68.84%	64.57%	65.95%	70.72%	69.74%	76.25%	N/A	67.23%
C	62.46%	68.86%	58.99%	59.38%	47.74%	N/A	N/A	61.86%
NR	77.45%	72.86%	77.48%	67.59%	75.42%	113.92%	N/A	75.24%
Total	71.58%	70.39%	69.95%	69.10%	75.61%	72.63%	26.39%	70.76%

1 - Rounded to the nearest whole year.

Source: Moody's Rated Defaulted Borrowers Revolving Lines of Credits (1985-2007)

Unlike previous results in Araten *et al*. (2001), we do not observe collared LEQ rising monotonically with time-to-default buckets, as the pattern is a hump shape peaking at 4 years to default: average LEQ of 28.4%, 40.3%, 44.8%, 47.8%, 54.0%, 52.1% and 14% or less than 1, 1, 2, 3, 4, 5 and greater than 5 years to default, respectively. While the pattern that emerges for Winsorized LEQ is montonically increasing in TTD bucket, in the case of raw LEQ, strict monotonicity does not obtain. Again, as previously discussed in Table 1, we obtain significantly positive rank correlations of time-to-default (as a continuous variable) and all versions of the LEQ factor, and KW tests confirm that median LEQ is different in at least one (discrete) time-to-default bucket. We can also gain an understanding of how EAD risk behaves by the rating and TTD segments by looking at the conditional distributions in Figures 3.1-3.6 and 4.1-4.6, Figures 3.1-3.6 and 4.1-4.6, respectively, for the case of collared LEQ. The shift in probability mass from 1 to zero is clear as we go from best to worse (longest to shortest) rating class (time-to-default). In line with previous results, there is little pattern in the volatility of EAD risk measures as measured by standard deviation, as seen by examining the grids in the right-hand column. For example, in the case of collared LEQ, volatility is completely non-monotonic and never far from the mean of 39.2%: standard deviations of 37.8%, 40.4%, 39.7%, 41.4% and 33.3% for ratings AAA-BBB, BB, B, CCC-CC and C, respectively. By TTD in Table 3, a similar non-pattern obtains for collared LEQ: standard deviations of 40.6%, 39.4%, 40.1%, 42.1% and 40.5% for 1, 2, 3, 4 and 5 years to default, respectively. Similar patterns obtain for Winsorized and raw LEQ, standard deviations that bounce wildly around the overall mean.

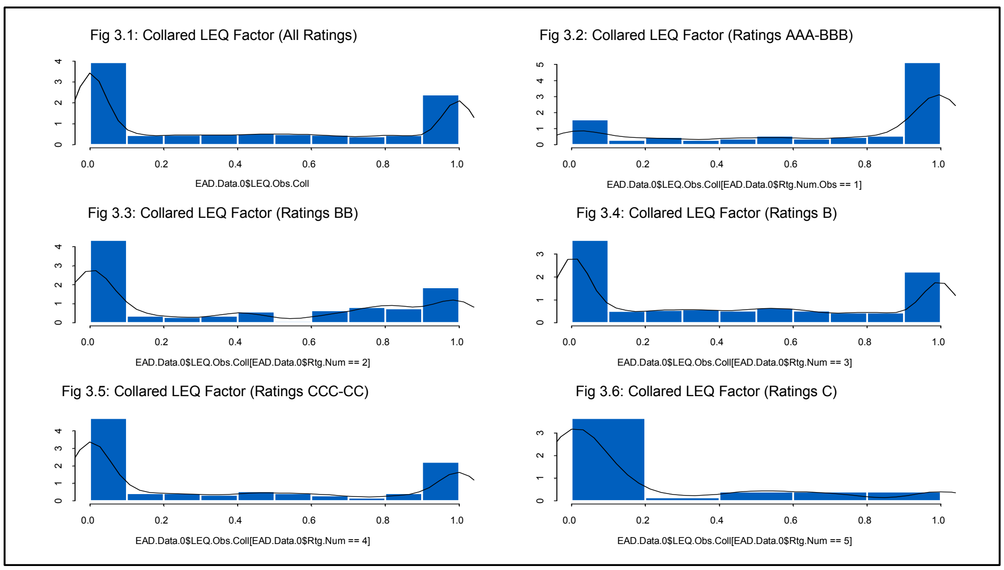

Figure 3. Distributions of CCF and EADF Factors by Rating

In the bottom 2 panels of Tables 3, averages of CCF and EADF both appear to be decreasing in risk rating, although the rate of decrease is weaker than for LEQ, and it is non-monotonic: for CCF (EADF) 336.8%, 174.9%, 191.4%, 158.2% and 115.6% (90.4%, 65.7%, 70.0%, 67.2% and 61.9%) for ratings AAA-BBB, BB, B, CCC-CC and C, respectively. In the case of CCF, the term structure appears to be almost monotonically increasing in TTD bucket (but actually an inverse u-shape with peak at 4 years): averages of 142.2%, 174.2%, 199.8%, 239.5%, 264.1%, 256.1% and 78.9 for less than 1, 1 through 5 and greater than 5 years to default, respectively. However, for EADF we see a nearly flat term structure: averages of 71.6, 70.4%, 70.0%, 69.1%, 75.6% 72.6% and 26.39% for less than 1, 1 through 5 and greater than 5 years to default, respectively.

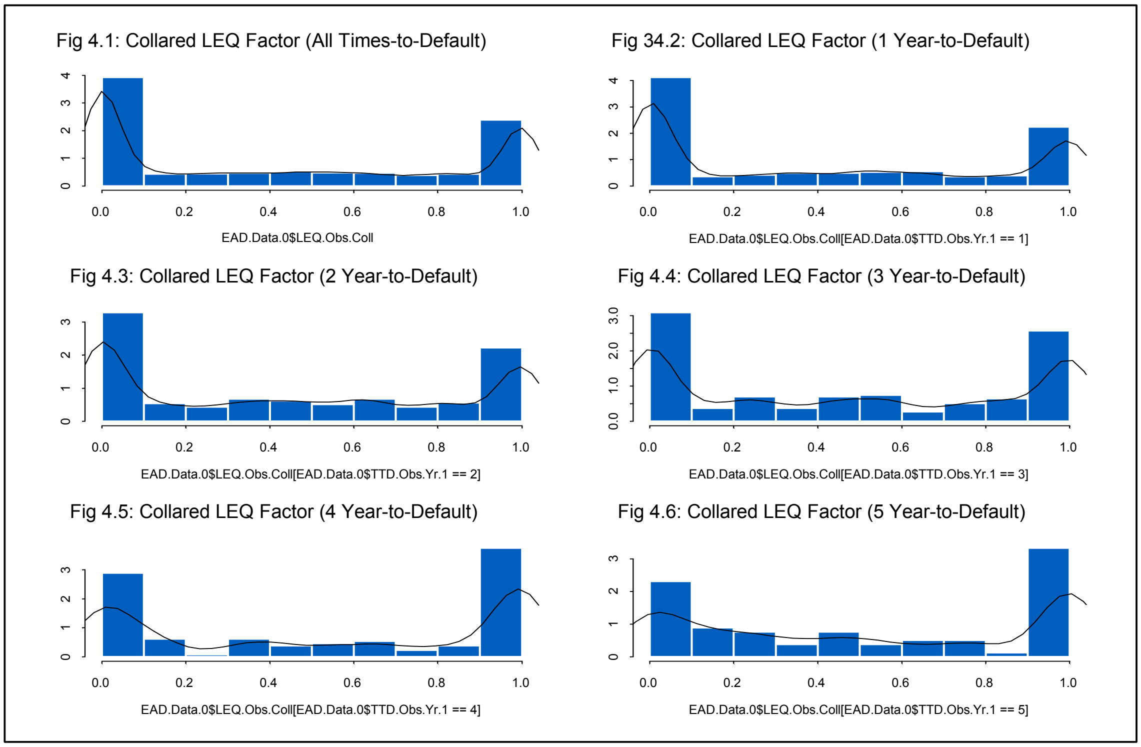

Figure 4. Distributions of CCF and EADF Factors by Time-to-Default

In Table 4, we tabulate the proportion of observations that fall into various regions of interest, for our three measures of EAD risk. This is done by major rating category and by TTD bucket. The values of interest include negative raw LEQ factors, which are indicative of pay-down behavior on the part of borrowers migrating to default, or values in excess of unity, indicative of additional extensions of credit (i.e., increases in line limits) on the way to default. In the case of raw CCF factors, we are interested in values greater than unity, as these reflect increases in balances on the way to default, which are in a sense perverse in that we would expect banks to be working balances down; although, borrowers would be intensifying their rates of drawdown as well in the face of limit shrinkage, so that values greater than one are also worth studying[30]. In the case of raw EADF, estimates in excess of one are also peculiar in some sense, as we would expect banks to be reducing line limits as obligors become increasingly distressed. In the case of raw LEQ, we observe that just under one-third of the observations (28.6%) are negative, and 14.2% are greater than 100%. We see that the propensity toward paydown behaviour by this measure does seems to be highly non-linear in rating, as it is much lower for the highest class (7.3% of observations), and then non-monotonic and little differentiated in this others. So it would seem that the best rated borrowers are actually the least likely to pay-down fractions of the unused from some point to default, which may be intuitive, as one might think that borrowers who have deteriorated would be more likely to pay down in order to placate lenders (or more susceptible to pressure from lenders to do such). But on the other hand it could be that higher rated entities have the ability to substitute other sources of financing, so in that sense this might just as plausibly be considered counter-intuitive. On the flip side of this, we see somewhat clearly that better rated obligors are able to effectuate eventual drawdowns in excess of previous unused amounts, and that this ability diminishes as they are down-graded. In the time dimension, perhaps counter-intuitively we observe more paydown behavior as default becomes imminent, as perhaps we would expect more financially strained borrowers to have less ability as well as willingness to do this. However, on the ability or tendency for drawdowns to eventually increase over previous undrawn amounts, it appears that this is working in the "right" direction, as we see monotonically fewer observations of this as time-to-default shrinks. Turning to raw CCF, there is rather a

[30] Some banks refer to CCF as "balance inflation factor" to reflect the expectation that there is an expectation that while limits are being reduced, at least on average we should see outstanding balances increasing as firms near a default state.

clear pattern of balance inflation with better rating category prior to default, perhaps in line with common wisdom that more lightly monitored obligors carrying better credit grades are able to more easily draw on banks, although the relationship is not monotonic. And possibly consistent with this, we generally see shrinkage in balances associated with downward credit migration. In time-to-default we also see something perhaps readily explainable, more (less) balance inflation (deflation) with lengthening tenor. Finally for Table 4, focusing in on raw EADF, it is evident that better rated obligors tend to experience increases in limits prior to default, and at the same time they are less likely to see reductions in limits. We observe generally more (less) limit inflation (deflation) with better rating. And perhaps in also intuitively, the longer the horizon to default, the more likely we are to observe either increases in limits, or the less prevalent are observations of decreases in such.

Table 5 documents the cyclical properties of the EAD risk measures, examining EAD risk measures by year of observation. We have identified the downturn periods as 1991-90 and 2001-2002, based upon these being the years having the highest Moody's 12-Month Trailing Speculative Grade Default Rates in the sample period; so, these years are conventionally considered as the downturn periods, and roughly coincide with the NBER peak and trough dates.[31] It is a little hard to see the credit cycle reflected in the counts of defaulted facilities in the data-set, which in the case of the LEQ factors peak at 61 in 1989 for the earlier downturn, and then at 271 in 2000 in the most recent downturn, appearing to lead the cycle. Similar patterns for the CCF and EADF.

It would appear that all of the EAD risk measures are elevated in the middle 1980's, 1990's and the middle of this decade - periods thought to be expansionary. However, low counts at the beginning and end of the sample warrant some caution in interpreting these numbers. Nevertheless, the results for the mid-1990's are reasonable robust, as we have ample observations in that period. First, in the case of LEQ, while the pattern is murky for the 1980's, we see clear elevation in the averages for the mid 1990's, as well as the years following the most recent downturn. Considering the 1st downturn, LEQ averaged 32.5% from 1985-1990 (peaking at 36.1% in 1989), yet higher in average at 37.8% during the 1991-1992 downturn. However, in the second cycle it seems that LEQs are elevated in the more benign period, averaging 46.2% in 1993-2000 (peaking at 54.3% in 1996), significantly higher than the average of 37.0% in 2001-2003 (having a local trough of 35.2% in that period); and then increasing into the middle this decade, averaging 38.5% in 2004-2007 (having a peak of 40.9% in 2004). However, the sample size is limited in the last 2 years of the data-set, so we have to be careful in how much weight that we are willing to place on this result. Restricting the sample to observations one year prior to default, in the 2nd and 3rd columns, yields qualitatively similar results. Next, in the case of CCF, while the pattern for the 1980's is now in line with the rest of the sample periods, as we see clear elevation then as well as the averages for the mid 1990's and the years following the most recent downturn. Considering the 1st downturn, CCF averaged 167.9% from 1985-1990 (peaking at 209.4% in 1987), and lower in average at 154.4% during the 1991-1992 downturn. In the second cycle it seems that CCFs are elevated in the more benign period, averaging 209.0% in 1993-2000 (peaking at 242.2% in 1998), seemingly higher than the average of 162.1.0% in 2001-2003 (having a local trough of 151.3% in year 2002 of that period); and then increasing into the middle this decade, averaging 176.3% in 2004-2007 (having a peak of 201.5% in 2005). As with the other parameters, the same caveat applies to the last years of the sample, and restricting the sample to observations one year prior to default (in the 6th and 7th columns) yields qualitatively similar results.

Table 4. Estimated Regions of LEQ, CCF and EAD Factors by Rating and Time-to-Default[1]

S&P and Moodys Rated Defaulted Borrowers Revolving Lines of Credits 1985-2007											
Raw LEQ											
Risk Rating	Region					Time –to-Default	Region				
	< 0	= 0	.(0,1)	=1	>1		< 0	= 0	.(0,1)	=1	>1
AAA-BBB	7,27%	1,82%	45,45%	16,36%	29,09%	1	30,42%	5,51%	45,44%	8,37%	10,27%
BB	32,00%	3,43%	52,00%	1,71%	10,86%	2	28,73%	0,81%	51,22%	5,15%	14,09%
B	27,49%	4,04%	50,32%	4,67%	13,49%	3	26,98%	0,47%	49,30%	5,12%	18,14%
CCC-CC	33,89%	9,30%	36,54%	6,31%	13,95%	4	21,09%	0,78%	48,44%	4,69%	25,00%
C	27,03%	18,92%	45,95%	2,70%	5,41%	5	16,67%	0,00%	52,56%	3,85%	26,92%
Total	28,63%	5,75%	45,26%	6,19%	14,16%	Total	28,63%	5,75%	45,26%	6,19%	14,16%
Raw CCF											
Risk	Region					Time –to-	Region				

[31] These are peak in July 1990 and March 2001, and troughs in March 1991 and November 2001.

Rating	< 0	= 0	.(0,1)	=1	>1	Default	< 0	= 0	.(0,1)	=1	>1
AAA-BBB	N/A	N/A	11,43%	2,86%	85,71%	1	N/A	N/A	33,76%	6,12%	57,17%
BB	N/A	N/A	38,36%	4,79%	56,85%	2	N/A	N/A	35,45%	1,00%	61,87%
B	N/A	N/A	33,69%	5,10%	61,21%	3	N/A	N/A	34,94%	0,60%	62,65%
CCC-CC	N/A	N/A	41,53%	11,29%	47,18%	4	N/A	N/A	29,03%	2,15%	66,67%
C	N/A	N/A	30,30%	21,21%	48,48%	5	N/A	N/A	31,71%	0,00%	65,85%
Total	N/A	N/A	34,14%	6,99%	56,32%	Total	N/A	N/A	34,14%	6,99%	56,32%
Raw EADF											
Risk	Region					Time –to-	Region				
Rating	< 0	= 0	.(0,1)	=1	>1	Default	< 0	= 0	.(0,1)	=1	>1
AAA-BBB	N/A	N/A	54,55%	16,36%	29,09%	1	N/A	N/A	84,15%	6,04%	9,81%
BB	N/A	N/A	86,93%	2,27%	10,80%	2	N/A	N/A	81,40%	8,35%	10,25%
B	N/A	N/A	81,74%	4,79%	13,48%	3	N/A	N/A	80,81%	5,14%	14,05%
CCC-CC	N/A	N/A	79,93%	6,25%	13,82%	4	N/A	N/A	76,74%	5,12%	18,14%
C	N/A	N/A	91,89%	2,70%	5,41%	5	N/A	N/A	69,77%	5,43%	24,81%
Total	N/A	N/A	79,58%	6,30%	14,11%	Total	N/A	N/A	79,58%	6,30%	14,11%

Source: Moody's Rated Defaulted Borrowers Revolving Lines of Credits (1985-2007)

Finally for EADF, we are back to murkiness for the 1980's, yet clear elevation in the averages for the mid 1990's and the years following the most recent downturn. Considering the 1st downturn, EADF averages 66.0% in the 1985-1990 period (peaking at 69.7% in 1989), yet higher in average at 77.0% during the 1991-1992 downturn. However, in the second cycle it seems that EADFs are elevated in the more benign period, averaging 73.3% in 1993-2000 (peaking at 83.6% in 1996), significantly higher than the average of 65.3% in 2001-2003 (having a local trough of 55.4% in year 2003 that period); and then increasing slightly into the middle this decade, averaging 65.9% in 2004-2007 (having a peak of 69.6% in 2004). Again, the low counts at the end of the sample warrants caution in interpreting these results. Restricting the sample to observations one year prior to default, in the 10th and 11th columns, yields qualitatively similar results.

Table 6 attempts to shed further light on the issue of downturn EAD by examining the time relationship by industry category. The industries High Technology/Telecommunications, Leisure Time/ Media and Transportation appear to have elevated EAD risk by respective LEQ factors 49.3%, 46.2% and 47.9%. However, according to the other measures these are not necessarily the riskiness industries. In the case of CCF, Energy / Natural Resources and Transportation are furthest above the overall mean of 190.4%, with average CCF factors of 203.9% and 215.5%, respectively. And in the case of the EADF factors, Insurance and Real Estate is the furthest above the mean, an average of 92.8%. Note that among these, Leisure Time / Media is the only industry to have an elevated average of Moody's Speculative Grade Default Rate of 7.1%, as compared to the overall average of 5.9%. While Utilities are rather EAD high risk according to the LEQ and CCF factors, having respective averages of 50% 233.9%, there are only 6 observations in this category. Forest / Building Products / Homebuilders appear rather to have rather low EAD risk according to the LEQ and CCF factors, with respective means of 29.0% and 126.2%, with EADF agreeing albeit not as strongly, an average of 61.0%; but with only 17 observations in that group, we have to be cautious in how we interpret these results. Also note that this industry does not seem to have elevated default risk, an average of Moody's Speculative Grade Default Rate of 5.8%, close to the overall average.

In Tables 7, we look at EAD risk measures by type of collateral, as well as the seniority (Senior, Subordinated and Junior Subordinated) of the loan. Note that we rely upon a ranking a collateral quality based upon aggregations of the lower level labels in the MURD database, developed in consultation with subject matter experts at Moody's. The ranking that we have developed is, in order, as follows: Cash/Guarantees/Other Highly Liquid Assets, Inventories/Receivables/Other Current Assets, Second lien/Real Estate/All Assets/Oil & Gas Reserves, Capital Stock/Inter-company Debt, Plant, Property & Equipment ("Collateral Rank 5").

Most Assets/Intellectual Property and Unsecured. Breaking the observations down by major collateral categories and three seniority ranks, there are some patterns that can be gleaned. These seem to imply that collateral may matter, as across all measures we generally observe that there seems to be greater EAD risk for less well secured, or unsecured, instruments. Average LEQ, CCF and EADF is 57.1%, 240.0% and 76.5% (41.3%, 187.9% and 70.4%) for unsecured (secured); although the counts of unsecured loans are rather low in the database (90, 64 and 91 for LEQ, CCF and EADF, respectively), so that we should be careful in how we interpret this pattern. However, we do observe that generally amongst the secured observations, EAD risk measures are lower, as we go up the collateral quality ranks. In the case of LEQ we proceed monotonically upward in average from Collateral Rank 1 to 6. The pattern is near-monotonically increasing in average for CCF and EADF. We see similar patterns within seniority ranks, although it is bumpier. We further observe that EAD risk measures are higher as we go down the seniority ranks. In the case of LEQ, the relationship is only approximately increasing, as the averages are 39.2%, 51.0% and 47.0% for creditor ranks 1 to 3. On the other hand, we have strict increase in the other EAD risk measures as we go to lower seniority ranks. Finally, we try to look at EAD risk measures by minor collateral category. Unfortunately, it is difficult to discern a consistent pattern across these for any of the measures.

Table 5. LEQ, CCF and EADF of Defaulted Instruments and Obligors by Observation Year

Year	Cnt, of LEQ	Avg. of LEQ[1]	Cnt. of LEQ I Yr. to Default	Avg. of LEQ[1] I Yr. to Default	Cnt. of CCF	Avg. of CCF[2]	Cnt. of CCF I Yr. to Default	Avg. of CCF[2] I Yr. to Default	Cnt. of EADF	Avg. of EADF[3]	Cnt. of EADF I Yr. to Default	Avg. of EADF I Yr to Default	Avg. of Rtg.[4]	Avg. of Util.[5]	Avg. of Commit.[6]	Moody's Spec. Grade Default Rate
1985	1	29,17%	1	29,17%	1	103,10%	1	103,10%	1	93,20%	1	93,20%	2,00	90,40%	250.000	4,10%
1986	4	15,68%	1	62,73%	4	103,63%	1	157,91%	4	71,30%	1	82,11%	3,00	77,02%	128.750	4,97%
1987	7	27,14%	3	25,97%	7	209,44%	3	315,93%	7	67,80%	3	46,48%	3,14	68,79%	97.741	5,79%
1988	22	27,16%	4	33,56%	21	203,18%	4	162,53%	22	56,57%	4	75,88%	2,89	57,51%	150.362	4,89%
1989	59	36,12%	32	24,93%	52	153,51%	27	119,09%	59	64,91%	32	59,17%	2,80	55,53%	134.264	2,74%
1990	61	31,76%	27	44,85%	59	167,52%	25	197,98%	62	69,73%	27	74,57%	2,93	62,31%	90.321	6,58%
1991	34	34,08%	11	21,90%	34	126,45%	12	87,17%	34	75,37%	11	63,20%	3,32	72,32%	102.156	12,09%
1992	32	41,83%	15	41,84%	31	185,09%	14	168,41%	32	78,72%	15	76,08%	2,95	62,68%	57.347	7,32%
1993	33	43,46%	10	53,50%	32	141,39%	10	140,85%	33	82,29%	10	81,52%	2,82	65,59%	74.686	5,06%
1994	44	39,01%	20	40,48%	42	199,40%	19	191,58%	44	77,22%	20	77,12%	3,14	57,34%	83.421	2,80%
1995	43	42,09%	14	52,99%	39	174,40%	13	153,29%	43	75,96%	14	81,30%	2,97	55,91%	114.915	2,06%
1996	44	54,34%	13	50,12%	38	218,06%	12	195,92%	44	83,63%	13	84,59%	3,03	46,95%	118.885	3,01%
1997	89	47,81%	16	49,03%	71	232,62%	15	191,95%	89	76,83%	16	91,95%	2,93	40,05%	180.207	2,24%
1998	205	51,34%	49	51,12%	162	242,20%	44	211,00%	205	76,61%	49	77,68%	2,83	38,78%	170.155	2,98%
1999	237	45,79%	47	39,46%	195	206,65%	46	199,12%	237	71,70%	47	71,01%	2,88	45,80%	178.425	4,58%
2000	271	42,83%	103	43,36%	204	194,02%	91	202,05%	271	67,16%	103	67,95%	2,88	44,39%	239.693	6,80%
2001	184	37,85%	64	39,86%	150	165,86%	50	164,16%	185	66,37%	64	68,60%	3,03	49,34%	308.868	9,13%
2002	95	35,19%	46	31,40%	86	151,30%	41	114,70%	98	65,03%	47	62,83%	3,32	53,80%	230.946	11,01%
2003	59	37,20%	26	31,78%	53	169,15%	25	136,51%	59	62,65%	26	55,40%	3,51	55,01%	102.979	6,83%
2004	33	40,94%	14	46,78%	27	168,12%	10	160,27%	33	65,95%	14	72,98%	3,52	44,81%	170.113	4,77%
2005	22	40,26%	8	57,52%	19	201,48%	8	219,20%	22	69,55%	8	81,87%	3,30	46,24%	57.202	2,94%
2006	2	0,00%	2	0,00%	2	88,07%	2	88,07%	2	31,44%	2	31,44%	4,00	56,76%	61.300	2,28%
2007	1	0,00%	0	0,00%	1	95,92%	0	0,00%	1	53,41%	0	0,00%	4,00	55,68%	44.000	1,63%
Total	1.582	42,21%	526	40,81%	1.330	190,42%	473	174,17%	1.587	70,76%	527	70,39%	2,99	48,64%	184.027	5,17%

1 - Loan Equivalent Factor
2 - Credit Conversion Factor
3 - Exposure at Default Factor
4 - Numeric codes for major ratings (on S&P scale): 1 =AAA-BBB, 2 = BB, 3 = B, 4 = CCC-CC, 5 = C
5 - Outstanding / Total Commitment
6 - Total legal commitment (millions)

Source: Moody's Rated Defaulted Borrowers Revolving Lines of Credits (1985-2007)

Table 6. LEQ, CCF and EADF of defaulted instruments and obligors by industry (S&P and Moody's Rated Defaults 1985-2007)

Industry Group	Cnt. of LEQ	Avg. of LEQ[1]	Cnt. of LEQ I Yr. to Default	Avg. of LEQ[1] I Yr. to Default	Cnt. of CCF	Avg. of CCF[2]	Cnt. of CCF I Yr. to Default	Avg. of CCF[2] I Yr. to Default	Cnt. of EADF	Avg. of EADF[3]	Cnt. of EADF I Yr. to Default	Avg. of EADF[2] I Yr. to Default	Avg. of Rating[4]	Avg. of Moody's Spec. Grd. Default Rate	Avg. of Util.[5]	Avg. of Commit.[6]
Aerospace / Auto / Capital Goods / Equipment	225	40,09%	62	38,46%	202	188,99%	60	194,47%	227	68,52%	63	70,63%	3,01	6,38%	48,88%	120.843
Consumer / Service Sector	428	36,61%	153	40,34%	374	186,30%	138	189,98%	428	67,73%	153	71,09%	3,02	6,12%	48,22%	138.039
Energy / Natural Resources	162	47,65%	47	38,27%	114	203,87%	40	161,46%	162	74,04%	47	63,47%	2,85	4,36%	40,06%	304.305
Financial Institutions	11	45,34%	5	50,49%	11	142,00%	5	137,89%	11	72,21%	5	71,93%	3,60	7,06%	52,87%	33.722
Forest / Building Products / Homebuilders	40	29,02%	17	25,73%	36	126,34%	16	103,91%	40	64,30%	17	60,98%	2,94	5,79%	55,83%	114.421
Healthcare / Chemicals	149	38,51%	50	37,84%	123	165,09%	45	169,58%	150	69,47%	50	73,46%	3,02	5,72%	47,75%	168.155
High Technology / Telecommunications	213	49,30%	77	45,06%	146	199,88%	53	137,99%	213	75,47%	77	71,20%	2,93	5,72%	37,59%	276.191
Insurance and Real Estate	17	35,96%	9	26,92%	17	119,01%	9	111,21%	17	92,80%	9	87,42%	3,13	4,47%	82,76%	137.190
Leisure Time / Media	167	46,15%	60	47,27%	136	178,73%	53	172,72%	167	72,21%	60	71,78%	3,17	7,09%	46,01%	150.574
Transportation	164	47,86%	43	41,44%	131	215,54%	38	171,30%	166	71,43%	43	68,03%	2,86	5,30%	42,24%	203.296
Utilities	6	50,00%	3	66,67%	6	233,87%	3	372,18%	6	67,16%	3	70,80%	2,50	5,27%	42,20%	233.267
Total	1.582	42,21%	526	40,81%	1.330	190,42%	474	173,91%	1.587	70,76%	527	70,39%	2,99	5,91%	48,64%	184.027

1 - Loan Equivalenct Factor
2 - Credit Conversion Factor
3 - Exposure at Default Factor
4 - Numeric codes for major ratings (on S&P scale): 1 =AAA-BBB, 2 = BB, 3 = B, 4 = CCC-CC, 5 = C
5 - Outstanding / Total Commitment
5 - Total legal commitment (millions)

Source: Moody's Rated Defaulted Borrowers Revolving Lines of Credits (1985-2007)

Table 7. EAD risk measures by instrument and major collateral types

		LEQ[2]				CCF[3]				EADF[4]			
		Senior - Creditor Rank 1	Sub. - Creditor Rank 2	Junior Sub. - Creditor Ranks 3-6	Total	Senior - Creditor Rank 1	Sub. - Creditor Rank 2	Junior Sub. - Creditor Ranks 3-6	Total	Senior - Creditor Rank 1	Sub. - Creditor Rank 2	Junior Sub. - Creditor Ranks 3-6	Total
Cash / Guarantees / Other Highly Liquid Assets	Count	28	7	0	35	24	5	0	29	28	7	0	35
	Average	17,73%	26,88%	N/A	19,56%	77,43%	204,74%	N/A	99,38%	44,62%	86,31%	N/A	44,53%
	Stdev	34,64%	24,36%	N/A	39,69%	49,01%	145,96%	N/A	86,09%	38,07%	8,08%	N/A	34,10%
Inventories / Receivables / Other Current Assets	Count	212	42	13	267	187	35	8	230	212	42	13	267
	Average	32,57%	56,42%	46,06%	36,98%	160,34%	255,43%	269,32%	178,60%	63,68%	86,31%	60,57%	67,09%
	Stdev	38,17%	43,63%	52,84%	39,69%	170,92%	260,54%	327,57%	192,58%	34,90%	260,54%	754,15%	192,58%
Second Lien / Real Estate /All-Assets / Oil & Gas Reserves	Count	719	229	96	1044	641	171	72	884	722	230	96	1048
	Average	38,02%	48,92%	44,33%	40,99%	172,37%	220,90%	221,61%	185,77%	69,14%	72,03%	73,65%	70,19%
	Stdev	40,10%	39,69%	44,42%	40,39%	178,90%	236,42%	252,36%	197,69%	36,68%	35,15%	40,06%	36,64%
Capital Stock / Inter-company Debt	Count	54	17	0	71	42	17	0	59	54	17	0	71
	Average	51,85%	44,79%	N/A	50,16%	150,78%	171,14%	N/A	156,64%	84,36%	71,60%	N/A	81,30%
	Stdev	44,51%	37,83%	N/A	42,85%	101,78%	187,55%	N/A	130,81%	27,89%	187,55%	N/A	27,43%
Plant, Property & Equipment	Count	15	0	0	15	9	0	0	9	15	0	0	15
	Average	N/A	0,00%	N/A	53,92%	N/A	0,00%	N/A	226,31%	65,68%	0,00%	N/A	65,68%
	Stdev	N/A	0,00%	N/A	35,73%	N/A	0,00%	N/A	130,29%	34,52%	0,00%	N/A	34,52%
Most Assets / Intellectual Property	Count	51	2	7	60	49	1	5	55	51	2	7	60
	Average	61,16%	98,70%	85,48%	65,24%	327,55%	429,82%	N/A	335,41%	88,71%	112,47%	113,84%	92,43%
	Stdev	43,15%	1,83%	39,26%	41,65%	304,83%	N/A	N/A	299,01%	34,94%	20,47%	932,15%	299,01%
Total Secured	Count	1079	297	116	1492	952	229	85	1266	1082	298	116	1496
	Average	37,68%	49,56%	53,98%	41,31%	173,02%	223,05%	260,18%	187,92%	69,07%	73,64%	74,61%	70,41%
	Stdev	40,66%	40,37%	44,17%	40,86%	185,05%	235,61%	257,29%	200,36%	36,54%	37,05%	41,67%	37,04%
Unsecured	Count	62	26	2	90	47	16	1	64	63	26	2	91
	Average	53,12%	67,51%	44,85%	57,09%	224,65%	292,02%	126,47%	239,96%	77,30%	75,74%	63,24%	76,54%
	Stdev	43,72%	37,73%	69,11%	41,99%	256,93%	260,93%	N/A	255,89%	34,21%	39,92%	20,17%	35,40%
Total Collateral	Count	1141	323	118	1582	999	245	86	1330	1145	324	118	1587
	Average	39,23%	51,01%	46,97%	42,21%	177,49%	227,55%	234,94%	190,42%	69,52%	73,81%	74,42%	70,76%
	Stdev	40,81%	40,11%	44,43%	40,92%	188,87%	236,76%	258,81%	203,38%	36,42%	37,29%	41,19%	36,94%

1 - 496 defaulted borrowers, having a revolving credit exposure prior to default, sampled at one year anniversaries or changes in risk rating prior to default

2 - Empirically measured Loan Equivalent Exposure where $LEQ_{t,T}$ = ($Drawn_T$ - $Drawn_t$)/$Undrawn_t$, T (t) = default (observation) floored (capped) at 0% (100%) = max(min(LEQ,1),0)

3 - Credit Conversion Factor: $CCF_{t,T}$ = $Drawn_T$/$Drawn_t$, T (t) = default (observation) date Winsorized at the 5th and 95th quantiles of the sampling distribution

4 -Exposure at Default Factor: $EAD_{t,T}$ = $Exposure_T$/$Exposure_t$, T (t) = default (observation) date.

Source: Moody's Rated Defaulted Borrowers Revolving Lines of Credits (1985-2007)

Table 8 tabulates the estimation results for the beta link generalized linear model (BLGLM) described in Section 4. Various sets of independent variables were analyzed in a series of univariate and multivariate regressions. The final set chosen for each EAD risk measure was determined based upon a partially quantitative, and partially judgmental, process that weighed the following (sometimes competing) considerations. First, measures of in-sample model performance, predictive accuracy (or goodness-of-fit) such as McFadden Pseudo-Rsquared (MPR2) and log-likelihood (LL), versus a Spearman rank correlation (SRC) measure of rank ordering (or discriminatory) accuracy. Second, we consider the signs and significance levels of independent variables. Finally, there is an attempt to find a parsimonious representation that has a large number of variables in common across models.

Table 8. Beta Link Generalized Linear Model Multiple Regression Models for EAD Risk Measures

S&P and Moodys Rated Defaulted Borrowers Revolving Lines of Credits (1985-2007)[1]						
	LEQ[2]		CCF[3]		EADF[4]	
	Partial Effect	P-Value	Partial Effect	P-Value	Partial Effect	P-Value
Utilization[5]	-0,3508	2,53E-06	-0,3881	6,52E-06		
Commitment[6]	3,64E-05	0,0723				
Cutback Rate[7]					-1,74E-03	0,0658
Drawn[8]			-0,0191	5,53E-07		
Undrawn[9]	3,27E-05	7,42E-03	2,20E-05	2,81E-06	7,45E-05	0,0441
Time-to-Default[10]	0,0516	1,72E-05	0,3462	1,58E-06	0,0225	2,08E-03
Rating 1[11]	-0,1442	0,0426	-0,2440	0,1015	-0,0503	0,1267
Rating 2[11]	-0,0681	6,20E-03	-0,1511	0,0835	-0,0093	0,3581
Rating 3[11]	-0,0735	1,03E-05	-0,1895	3,70E-03	-0,0079	0,0634
Rating 4[11]	-0,0502	2,08E-04	-0,1591	0,0977	-0,0135	0,0910
Rating 5[11]	-0,0110	0,1003	-0,0277	0,2278	-0,0068	0,1195
Leverage 1[12]	-0,0515	0,0714	-0,1332	0,0276		
Leverage 2[13]					-0,0922	0,0065
Size[14]	0,1154	2,63E-03	0,1855	0,0655	0,0463	0,1081
Intangibility[15]	0,0600	0,0214			0,0483	0,0878
Liquidity[16]	-0,0366	0,0251	-0,1110	0,0845	-0,0264	0,0960
Profitabilty[17]	-6,59E-04	0,0230	-5,79E-04	0,0265	-7,46E-05	0,0996
Colllateral Rank[18]	0,0306	3,07E-03	0,0816	0,0277	0,0111	0,1027
Debt Cushion[19]	-0,2801	5,18E-06	-0,5193	0,0122	-0,3073	7,34E-06
Speculative Default Rate[20]	-0,9336	0,0635	-0,0928	0,0960	-0,1766	5,03E-04
Percent Bank Debt[21]	0,2854	5,61E-06	0,3859	0,0928	0,3868	8,09E-03
Percent Secured Debt[22]	0,1115	2,65E-03			0,1830	2,71E-03
Degrees of Freedom	455		457		456	
Likelihood Ratio P-Value	7,48E-12		1,66E-19		7,62E-09	
Pseudo R-Squared	0,2040		0,2336		0,1611	
Spearman Rank Correlation	0,4670		0,5618		0,4115	
MSE of Forecasted EAD	2,74E+15		7,53E+15		2,23E+17	

1 - Defaulted borrowers, having a revolving credit exposure prior to default, sampled at one year anniversaries or changes in risk rating prior to default

2 - Empirically measured Loan Equivalent Exposure where $LEQ_{t,T}$ = ($Drawn_T$ - $Drawn_t$)/$Undrawn_t$, T (t) = default (observation) date floored (capped) at 0% (100%) = max(min(LEQ,1),0)

3 - Credit Conversion Factor: $CCF_{t,T}$ = $Drawn_T$/$Drawn_t$, T (t) = default (observation) date floored (capped) at the 1st (99th) percentiles = max(min(LEQ,516.95%),-874.57%)

4 -Exposure at Default Factor: $EAD_{t,T}$ = $Exposure_T$/$Exposure_t$, T (t) = default (observation) date

5 - Utilizationt = Drawnt/Commitmentt where Commitmentt = Drawnt + Undrawnt

6 - $Commitment_t$ = Total legal commitment or limit on credit line at observation date t ($000s)

7 - Percent change in drawn rom date prior until observation date

8 - $Drawn_t$ = Total amount outstanding on line at time t

9 - $Undrawn_t$ = Total undrawn commitment (legal commitment minus drawn) on line at time t

10 - Time from observation date to default of revolving credit (years)

11 - Numeric codes for major S&P rating: 1 = AAA-BBB, 2 = BB, 3 = B, 4 = CCC-CC, 5 = C

12 - Leverage measured by the ratio of long term debt to the market value of equity observed at 1 and 2 years prior to default

13 - Leverage measured by the ratio of total debt to the book value of assets observed at 1 and 2 years prior to default

14 - Company size measured by the logarithm of book value observed at 1 and 2 years prior to default

15 - Intangibility of assets measured by the ratio of intangible to total assets observed at 1 and 2 years prior to default

16 - Liquidity measured by the current ratio (current assets to current liabilities) observed at 1 and 2 years prior to default

17 - Profitability measured by the profit margin (ratio of net income to net sales) observed at 1 and 2 years prior to default

18 - Ranking of collateral quality

19 - Proportion of debt in the capital structure subordinated to the instrument.

20- Moody's trailing 1-year default rate calculated for quarterly cohorts of speculative grade rated issuers

21 -Proportion of bank debt in the capital structure

22 -Proportion of secured debt in the capital structure

Source: Moody's Rated Defaulted Borrowers Revolving Lines of Credits (1985-2007)

Results in Table 8 are broadly in line with the univariate analysis, and generally consistent across EAD risk measures. The strongest result that emerges, generally ranking highest in statistical significance (in terms of p-values, or PVs) as well as partial effect (PEs) magnitudes, is the inverse relationship between UTIL (percent utilization) and two of the EAD risk measures: PEs (PVs) of -0.35 and -0.39 (2.5E-06 and 6.5E-6) for LEQ and CCF, respectively. However, UTIL does not enter the EADF model; rather the CR (cutback rate) enters this model and not the others, having a marginal PE of -0.02 and PV of 0.07. While all parameters are directly related with the UNDRN (the undrawn amount in dollars), the partial effects are all rather small, and significance is only marginal in the case of EADF: PEs (PVs) of 3.3E-5, 2.2E-5 and 7.5E-5 (7.4E-3, 2.8E-6 and 0.04) for LEQ, CCF and EADF, respectively. On the other hand, the DRWN (the drawn amount in dollars) only enters the model for CCF, and while highly significant (PV of 9.17E-7), the economic significance is questionable (PE of only -0.02); this too is in line with the negative univariate Spearman correlation. However, LEQ had a reasonably sized negative correlation with DRAWN in that analysis, but this did not enter the regression model; while the correlation with EADF was the opposite sign, it was rather small.

Let us know consider the key variables of the Araten *el al.* (2001) study, the TTD (time-to-default measured in years) and ORR (the obligor risk rating measured as dummy variables). We see consistently across models that TTD is directly associated with EAD risk, but that the relationship is stronger for LEQ and CCF, having much higher PEs (0.05 and 0.35, respectively) and much lower PVs (1.7E-5 and 1.6E-6, respectively), as opposed to EADF, having a PE (PV) of only 0.02 (0.10) and in fact being nearly insignificant. ORR has negative coefficient estimates across all EAD risk measures. However, in some instances they are only marginally or not statistically significant, such as rating CCC-CC in the CCF model (PV = 0.23), or ratings AAA-BBB and BB in the EADF model (PVs = 0.13 and 0.36, respectively). Generally, the pattern in the magnitudes of the partial effects is decreasing as ratings worsen, albeit non-monotonically.

Now let us consider financial ratio variables. Five of six dimensions of the ratios from the univariate analysis survive in the multivariate regressions (measures of leverage, size, liquidity, intangibility and profitability), and have signs consistent with such across all 3 models; whereas a measure of cash flow does not enter any of the models. LTD/MVE, or leverage as measured by the ratio of long-term debt to the market value of equity, is negatively related to EAD risk and at least marginally significant in the LEQ and CCF models: PEs of -0.05 and -0.13 (PVs of 0.07 and 0.03), respectively. However, in the EADF model LTD/BVE, the accounting measure of leverage (long-term debt ratio to book value of total assets) enters, having a PE of -0.09 and a PV of 0.01. This is in line with the univariate results, and consistent with our hypothesis that more highly levered firms may be under closer scrutiny, and hence less able to draw down on unused lines. The BVTA measure of company size (the logarithm of the book value of assets) has positive coefficients across all models (PEs of 0.12, 0.19 and 0.05 for LEQ, CCF and EADF, respectively); however, it is only highly statistically in one of the models (PE = 2.6E+4 for LEQ), marginally significant in another model (PE = 0.07 for CCF) and just short of significant in the third model (PE = 0.11 for EADF).

This result is what we saw in the correlation analysis, and may be explained by a tendency of banks to monitor larger companies less intensively, as they may be perceived as less likely to require use of their lines. The CR liquidity measure, as in the univariate analysis, is consistently negative across models (PEs of -0.04, -0.11 and -0.03 for LEQ, CCF and EADF, respectively), in line with the univariate correlation analysis; but it is marginally significant in 2 of the models (PVs of 0.085 and 0.096 in the CCF and EADF models), and only significant at the 5% level in the LEQ model (PV = 0.03). As alluded to before, this may be considered a reasonable result, as less liquidity constrained firms may draw less aggressively on their lines as they approach distress. Similarly, the PM (profit margin) profitability measure is consistently negative across models (albeit with small PEs of -6.6E-4, -5.8E-4 and -7.3E-5 for LEQ, CCF and EADF, respectively), in line with the univariate correlation analysis, and the expectation that less unprofitable firms on their way to default may pose lower EAD risk; but it is significant at the 5% level in only 2 of the models (PVs of 0.02 and 0.03 in the LEQ and CCF models), being just marginally significant in the EADF model (PV = 0.10). Finally for the financials, the INTA measure of intangibility (ratio of intangible to total assets) enters only the LEQ and EADF models, having positive PEs (0.06 and 0.05, respectively), as well as moderate significance levels (PVs of 0.02 and 0.05, respectively).

Now let us discuss results regarding measures of instrument-level characteristics. The COLL measure of collateral quality is present in all, and the CRED measure of seniority in none, of the regressions, consistent with the larger univariate correlations observed in the former as opposed to the latter. While the signs of the coefficient estimates for COLL are positive across models (PEs of 0.03, 0.08 and 0.01 for LEQ, CCF and EADF, respectively), only for LEQ do we observe high significance (PV = 3.1E+03), while for CCF significance is just at the 5% level (PV = 0.03), and for EADF we are just shy of significance at the 10% level (PV = 0.103). Second, the CUSH measure of tranche safety attributable to the revolving credit is inversely related to EAD risk across regression models, as in the univariate analysis. In this case, PEs are relatively strong, as compared to some other variables: -0.28, -0.52 and -0.31 for LEQ, CCF and EADF, respectively. In this case, significance levels are also notably high, at much better than the 1% level in 2 cases (PVs of 5.2E-6 and 7.3E-6 for LEQ and EADF, respectively), and at the 5% level in another (PV = 0.03 for CCF). These results suggest that while superior collateral does seem to mitigate EAD risk, above and beyond this there is a beneficial effect to be had from greater debt cushion.

Among capital structure variables, only the PERCBNK (percent bank debt) and the PERCSEC (percent secured debt) enters the leading regression models. In the case of PERCBNK, which enters all models, coefficient estimates are economically significant and of positive sign (PEs of 0.29, 0.39 and 0.39 for LEQ, CCF and EADF, respectively), in line with the observed correlations. However, while highly statistically significant for LEQ and EADF (with PVs of 5.6E-6 and 8.1E-3, respectively), this variable barely achieves such status in the CCF model (PV = 0.09). Nonetheless, this result has a rationale in a story that when more banks are present in the creditor group, there may be coordination problems (e.g., this may be associated with a larger syndicate). Alternatively, through the economic incentives of banks at the top of the capital structure, the optimal foreclosure boundary may be set higher than otherwise (Carey, and Gordy, 2007), and to the extent that lower LGD rates may be associated with this, an inverse correlation with EAD (if we believe that a tradeoff exists) may be consistent with the empirical result that we are finding. On the other hand, PERCSEC appears in only two models, LEQ and EADF, significantly (PVs of 0.03 in both) and having positive signs (PEs of 0.11 and 0.18, respectively).

The final variable that we consider is the measure of the economic cycle that made it into the final regression models, the MSG12MTDR (speculative default rate). This is expected, as the univariate Spearman correlations for MSG12MTDR were generally higher than for MAC12MTDR and SPR across all EAD risk measures[32]. We have evidence of counter-cyclicality, as all partial effects are negative (-0.93, -0.09 and -0.18 for LEQ, CCF and EADF, respectively). However, in only the EADF model do we have a high degree of statistical significance (PV = 50E-4); whereas in the LEQ and CCF models, significance is marginal (PVs of 0.064 and 0.96. respectively). Therefore, we can regard this as only limited evidence against “downturn EAD” or for counter-cyclicality in EAD risk. If we are willing to put some stock in these results, what economic rationale could be put forward? We could ascribe this to an “LGD-EAD” tradeoff: as the cycle turns downward and banks anticipate both higher default and higher recovery risk, they clamp down on revolving credit exposures, thereby reducing EAD even as loss severities may be rising.

Finally, we discuss in-sample measures of model quality in Table 8, measures of predictive accuracy (Log-Likelihood Ratio - LLR and McFadden Pseudo R-Squared - MPR2), discriminatory power (Spearman Rank correlation between actual and predicted values - SRC) and in-sample forecasting ability for dollar EAD (Mean Squared Error of forecasted dollar exposure at default, or MSE-EAD). The LLR and MPR2 are standard

[32] The other 2 measures considered in the correlation analysis, *Moody's All-Corporate Trailing 12-month Default Rate* (“MAC12MTDR”) and monthly return on the Standard & Poor’s 500 equity index return (“S&P Return” or “SPR”), were not significant in any of the regression models.

diagnostics assessing in-sample fit in non-linear models[33], having potentially non-normal errors[34]. The SRC here is meant to mimic the Area under the Receiving Operating Curve (AUROC) or Accuracy Ratio (AR) statistics calculated in binary dependent models, such of probability of default (PD) prediction, and is in fact a generalizationation of the concept.[35] The MSE-EAD measure is a bit non-standard, in that instead of focusing on how the models can predict or rank order the EAD risk measures, we focus on how the predicted parameters can forecast dollar EAD.[36]

Indeed, in the final specifications we observe that most variables are statistically significant, although it is about evenly split between very high levels of significance, and in some cases only marginally significance. However, there is much variation amongst in-sample performance measures, and we see in Table 8 that by these the CCF model performs best, and the EADF model ranks worst: the former model has the highest MPR2 of 0.30 (vs. 0.20 and 0.16 for LEQ and EADF, respectively) and highest SRC of 0.56 (vs. 0.47 and 0.41 for LEQ and EADF, respectively), as well as the smallest p-value on the likelihood ratio test of 1.7E-19 (vs. 7.5E-12 and 7.6E-9 for LEQ and EADF, respectively). However, in the exercise of forecasting the dollar EAD based upon these measures, in terms of MSE-EAD the LEQ measure performs best (2.7E+15), followed closely by CCF (7.5E+15), while EADF performs far worse than the other two (2.2E+17), the latter underperforming by about a factor of 100.

Table 9 and Figures 5-6 present the results of an out-of-sample and out-of-time analysis, which serves as a rigorous validation of the models estimated herein. The bootstrap exercise proceeds as follows: in the first set of cohort years constituting about half the sample (1985-1997), observations are chosen at random with replacement, and the models are estimated. Then the predictions of the models are evaluated in the next year (1998), upon a similarly chosen sample, and this is repeated several times (10,000 replications). Next, a year is added to the estimation s set (1998), and this is repeated, predicting EAD risk in bootstrapped 1999 samples based upon bootstrapped samples in the sub-set of years 1987-1998. This is done for 10 years (1998 through 2007), and the results are pooled to create 100,000 out-of–time and out-of-sample observations. The distributions of two model performance statistics, the MPR2 and SRC, are then studied. Table 9 shows some basic distributional statistics for these measures, while the full distributions of these are compared in Figures 5 and 6 (for MPR2 and SRC, respectively).

We observe that the ordering of the model performance in-sample is not preserved out-of-sample and out-of-time, as now the CCF model is not best by both measures, but now (by looking at the medians of the bootstrapped distributions) LEQ is slightly better by MPR2 (0.18 vs. 0.17), and CCF far superior by SRC (0.42 vs. 0.35). However, the distributions of these statistics shifted the furthest to the left for the EADF model and it remains ranked consistently last, having medians of 0.11 and 0.31 for MPR2 and SRC, respectively. However, upon examination of the low and high percentiles from Tables 8, as well as the range of the distributions in Figures 5 and 6, we observe that performance in the CCF model also exhibits the greatest amount of variation out-of-sample for both statistics, with numerical standard errors of 0.05 and 0.09, as compared to 0.03 and 0.07 (0.03 and 0.06) in the case of LEQ (EADF), for MPR2 and SRC, respectively.

Table 9. Bootstrapped[1] Out-of-Sample and Out-of-Time Classification and Predictive Accuracy Model Comparison Analysis of EAD Risk Measures

S&P and Moodys Rated Defaulted Borrowers Revolving Lines of Credits (1985-2007)[2]				
Test Statistic	**Model**	**LEQ[3]**	**CCF[4]**	**EADF[5]**
McFadden Pseudo R-Squared	Median	0,1839	0,1684	0,1084
	Standard Deviation	0,0255	0,0454	0,0260
	5th Percentile	0,0826	0,0291	0,0329
	95th Percentile	0,4151	0,3453	0,5942

[33] The MPR2 is a version of the standard linear r-squared typically seen in binary or qualitative variable models. While we have continuous variables here, we are dealing with bounded domains; hence the interpretation of the residuals suffers from similar issues. Therefore, we cannot interpret this measure as we would in an OLS setting.

[34] A model may be asymptotically consistent, albeit no efficient, if error are non-normal and we have a quasi-MLE interpretation, hence the LLR statistic. This is probably the case with these EAD risk measure.

[35] These statistics are in fact mathematically related to both the SRC, as well as the Wilcoxon Statistic (WS) for testing the equality of medians across distributions. It can be shown the WS is actually equivalent to the SRC between a binary dependent variable and a covariate or set of covariates.

[36] For example, estimated EAD prior to default would equal the outstanding at some horizon to default (i.e., a year), plus estimated LEQ (based upon the known values of explanatory variables at that time and the regression relationship) times the unused amount, and similarly for CCF and EADF.

S&P and Moodys Rated Defaulted Borrowers Revolving Lines of Credits (1985-2007)[2]				
Test Statistic	Model	LEQ[3]	CCF[4]	EADF[5]
Spearman Rank Correlation	Median	0,3461	0,4218	0,3078
	Standard Deviation	0,0676	0,0887	0,0642
	5th Percentile	0,2021	0,2427	0,1790
	95th Percentile	0,4865	0,5997	0,8224

1 - In each run, observations are sampled randomly with replacement from the training and prediction samples, the model is estimated in the training sample and observations are analyzed in the prediction period, and this is repeated 100,000 times

2 - Defaulted borrowers, having a revolving credit exposure prior to default, sampled at one year anniversaries or changes in risk rating prior to default

3 - Credit Conversion Factor: $CCF_{t,T}$ = $Drawn_T/Drawn_t$, T (t) = default (observation) date floored (capped) at the 1st (99th) percentiles = max(min(LEQ,516.95%),-874.57%)

4 -Exposure at Default Factor: $EAD_{t,T}$ = $Exposure_T/Exposure_t$, T (t) = default (observation) date.

5 - Utilizationt = Drawnt/Commitmentt where Commitmentt = Drawnt + Undrawnt

Source: Moody's Rated Defaulted Borrowers Revolving Lines of Credits (1985-2007)

Examining the distributions in Figures 5-6 in conjunction with the high / low quantiles in Table 9 tells a more complete story. First considering MPR2, we see that the distribution for LEQ is closest to bell-shaped, but the other two are multi-model and have longer right tails. Indeed, while CCF has the highest numerical standard error, and the numerical standard errors of EADF is almost the same as LEQ, we see that in many states of the world EADF performs better by the MPR2 measure (the 95th percentile is 0.59, 50% more than for LEQ). We see a similar comparison for SRC, that while EADF has the lowest measure of central tendency and numerical standard error, it has a right tail far elongated relative to LEQ or CCF.

The surprising shapes of these distributions highlights the dangers of using finite sample approximations to deriving test statistics for the model performance measures. Indeed, this implies that in 1 out of every 20 years of using this model on a holdout sample and re-rebuilding annually (this exercise being meant to mimic the way a practitioner would implement such models), MPR2s would fall into the single digits and SRCs would be around 20% for all models[37], which by industry standards would be considered dismal performance.

5. Conclusion

We have empirically investigated the determinants of, and built predictive econometric models for, measuring the risk associated with exposure at default (EAD). This has been accomplished through defining several metrics, which in principle should all give the correct answer: the loan equivalent exposure (LEQ), credit conversion factor (CCF) and the exposure at default factor (EADF). It has been shown that while they possess different numerical properties, as we must collar the LEQ and Winsorize the CCF in order to obtain sensible results, they share many of the same determinants. This has been illustrated using a sample of defaulted firms having revolving credits, at one time having S&P or Moody's rated debt. This builds upon a limited literature, as we have largely focused upon extending the prior empirical work of Araten *et al.* (2001) and Asarnow *et al.* (1994). This has been accomplished through incorporating borrower accounting, industry / macroeconomic and debt / equity market determinants of EAD, in addition to the traditional factors of credit rating, utilization and tenor. These various measures of EAD risk so derived have been compared through a multiple regression model, the so-called beta link generalized linear model (BLGLM) framework, a generalization of the well-known logistic modeling technology. As part of this exercise, we have examined the comparative rank ordering and predictive accuracy properties of these measures. We find weak evidence of counter-cyclicality in EAD risk, as these measures appear inversely related to the Moody's 12-month trailing speculative grade default rate, in both the univariate and the multiple regression analyses. We have confirmed many of the prior empirical results, as well as stylized facts, that EAD risk decreases with default risk, as risk rating has some explanatory power.

[37] This would correspond to an Accuracy Ratio of about 60%, which is very poor performance.

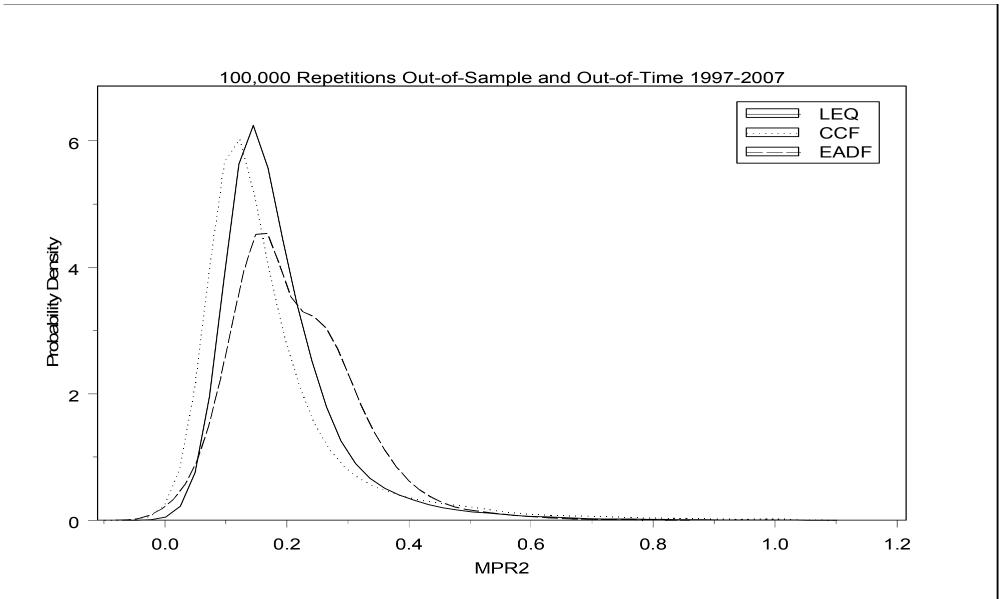

Figure 5. Densities of Pseudo-Rsquareds for EAD Prediction

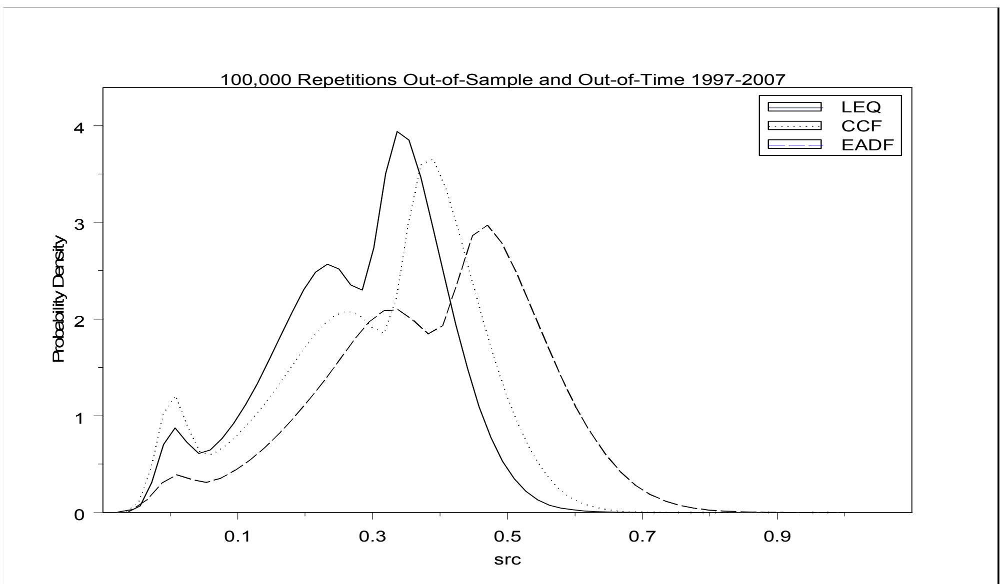

Figure 6. Densities of Spearman Rank Order Correlations for EAD Prediction

However, utilization has the strongest inverse relation with EAD risk by all three measures. We also find EAD risk reduced for greater leverage, augmented liquidity, or more debt cushion. Conversely, such is increased for greater company size, higher collateral rank of the loan or more bank debt in the capital structure of the defaulted obligor. The model is validated rigorously through resampling experiment in a rolling out-of-time and out-of-sample framework, in which we find the same relative ranking amongst EAD risk measures with respect to discriminatory power and predictive accuracy, where the EADF factor performs best, the CCF factor worst and the LEQ factor somewhere in the middle.

There exist various fruitful avenues along which this research may be extended. One option is to augment the data-set, to encompass either a larger sample of large-corporate revolving credits (e.g., an expanded cross-section), or to alternative instruments (such as trade or financial letters of credit). Another strategy would be the development of a more general framework, which would potentially encompass the three factors derived in this paper. Alternatively, we could pursue econometric designs better capable of dealing with outliers, such as robust/resistant regression techniques. We could also consider designing a theoretical model of EAD, wherein the parameter restrictions or functional forms so derived could be subject to empirical falsification. This could lead to joint estimation of EAD with PD or LGD risk measures.

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